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SHEAR MODULUS OF A FIBER COMPOSITE WITH A TRANSTROPIC VISCOLELASTIC MATRIX AND TRANSTROPIC ELASTIC FIBER

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When solving the problems of deformation solid mechanics, the inhomogeneous composite material is modeled as homogeneous, with averaged mechanical properties – effective characteristics. The purpose of this paper is to develop a technique for determining the effective shear modulus for a viscoelastic fiber composite with a transtropic matrix and fiber. Their isotropy planes coincide and are perpendicular to the fiber axis. The effective shear modulus is defined as a function of the matrix and fiber mechanical properties and the volume content of each of them in a composite. A unidirectional composite material with a hexagonal fiber stacking scheme and a unit cell consisting of a viscoelastic matrix and elastic fiber is considered. The geometric model of a composite is a combination of two coaxial infinite cylinders: a hollow cylinder, modeling the matrix, and a solid one, modeling the fiber inserted into it. The volume of the hexagonal cell is approximated by the volume of the cylinder. The radius of the cylinder is chosen so that the fiber volume content in the hexagonal cell coincides with the value of this characteristic for the cylindrical cell. To describe the viscoelastic properties of a composite, the ratios of the hereditary Boltzmann-Volterra theory are used. The shear modulus is defined as an integral operator with a difference kernel. Two boundary problems are considered: with regard to the longitudinal shear of a transonic viscoelastic solid cylinder modeling the composite, and the joint longitudinal shear of the hollow and solid cylinders that model the matrix and fiber materials, respectively. It is assumed that the displacements and tangential stresses on the contact surface of the matrix and fiber are continuous. A tangential harmonic load is applied on the outer surface of the cylindrical cell. To solve such problems, the Laplace transform is used. As the matching condition, the equality of displacements on the outer surface of the cylinder is used for the two problems. The application of the proposed technique makes it possible to determine the characteristics of the integral operator describing the shear modulus for a viscoelastic composite material. An instantaneous shear modulus and relaxation core parameters are found as the functions of the known mechanical characteristics of the matrix and fiber. As an example, the characteristics of the shear modulus for a composite material consisting of a rubber matrix and polyamide fiber are determined.

Keywords: *fiber composite material, effective shear modulus, viscoelasticity, transtropic material.*

1. Introduction

A technique is proposed for determining the shear modulus of a viscoelastic fiber composite with a transtropic matrix and fiber, depending on their mechanical characteristics, as well as the volume fraction of each of them in a composite. The case of a viscoelastic matrix and elastic fiber is considered. The isotropy planes of the matrix and fiber coincide and are perpendicular to the axis of the fiber. In order to obtain the characteristics of the integral operator determining the desired shear modulus two boundary value problems are considered: with regard to the longitudinal shear of a transonic viscoelastic solid cylinder modeling the composite, and the joint longitudinal shear of the hollow and solid cylinders that model the matrix and fiber materials, respectively. As the matching condition, the equality of axial displacements on the outer surface of the cylinder modeling said condition is used.

One of the most common methods of obtaining the mechanical characteristics of a composite material is the homogenization procedure, where the composite is represented as a homogeneous anisotropic material with the mechanical characteristics that depend on the mechanical characteristics of the matrix and reinforcing fibers, as well as the volume fraction of fibers in the composite. At that, it is believed that the frequency of reinforcement by the fibers is large enough, and the reinforced layer can be considered as transtropic. Then, to determine the mechanical characteristics of the composite, it is necessary to find five independent quantities: the elastic moduli E_{11} and E_{22}, the shear moduli G_{12} and G_{23}, and the Poisson ratio ν_{12}.

Such relationships for a composite material with an elastic transtropic fiber and isotropic matrix for the three-dimensional case were obtained in [1]. In [2], for the composite material the shear modulus G_{12} is determined, taking into account the trans-structural properties of the matrix and fiber. As the matching condition, the equality of the corresponding displacement vector components was used. The composite shear modulus taking into account the trans-structural properties of the matrix and the fiber was found in [3] on the basis of the application of the energy matching criterion. The problems of determining the mechanical viscoelastic characteristics of unidirectional composites based on the known characteristics of the matrix and fiber are considered in a number of works. In [4, 5], problems of predicting the viscoelastic properties of composites in the presence of a viscoelastic matrix or viscoelastic fiber are investigated, and the case of the presence of viscoelastic properties of both the matrix and fiber is also studied. The problem of determining the viscoelastic deformation characteristics of composites using the Boltzmann-Volterra hereditary theory of viscoelasticity is considered in [6]. There, a technique for determining the effective viscoelastic characteristics of composites, based both on the approximation of the deformation function by a continued fraction and application of the method of operator continued fractions. In [7], the homogenization of a two-component periodic composite with components separated by a viscoelastic layer is presented. To solve this problem, Green's function method is applied. In [8], a finite element analysis of the micromechanical model of a unidirectional fiber polymer composite under load is performed. At that, a viscoelastic matrix and elastic fiber are considered, and the matrix is assumed to have cracks.

In this paper, on the basis of the kinematic matching conditions, the shear modulus G_{12} of a composite consisting of transtropic elastic fibers and a transtropic viscoelastic matrix is determined.

1. Basic assumptions and relations

Consider a unidirectional composite material with a hexagonal fiber stacking scheme. From the composite volume, we cut out an elementary hexagonal cell containing one fiber and the surrounding matrix material. Let the matrix and fiber be made of transtropic materials with coincident isotropic planes perpendicular to the fiber axis. Suppose that the fiber material is elastic, and the matrix material is viscoelastic. Such properties are characteristic, for example, of rubber-cord materials.

As a rule, to describe the viscoelastic properties of materials, differential equations (based on the Hooke and Newton laws) or integral relations are used. The latter, more general, include the relations of the hereditary Boltzmann-Volterra theory, where viscoelastic properties are described by an integral operator with a difference relaxation kernel. The properties of a particular material are determined by the values of rheological characteristics included in the integral operator.

As a geometrical model of a composite, a combination of two coaxial infinite cylinders is considered – a hollow cylinder with an outer radius $r = b$, modeling the matrix and a solid one inserted into it with a radius $r = a$, modeling the fiber. The volume of the hexagonal cell is approximated by the volume of the cylinder, the cylinder radius being taken such that the volume fraction of the fiber $f = a^2/b^2$ in the hexagonal cell coincides with the value of this index for the cylindrical cell.

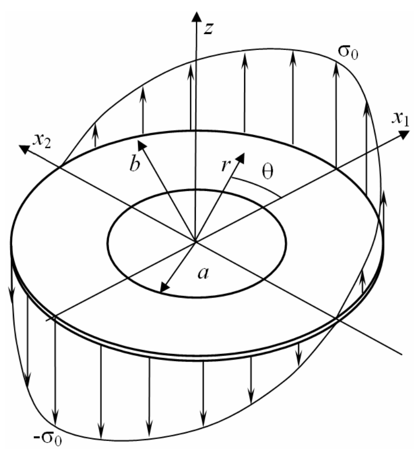

Fig. 1. Kind of load applied to the outer surface of the cylinder modeling the composite cell

The mechanical characteristics are determined from the solution to two boundary value problems. First, the problem of joint deformation of a transtronic viscoelastic matrix and transtronic elastic fiber is solved. As a result, we find the components of the stress-strain state (SSS) as a function of the mechanical characteristics of the matrix and fiber, as well as the volume fraction of the fiber in the composite.

Next, we obtain a solution of an analogous boundary value problem for a composite which is considered as a homogeneous viscoelastic transonic material with unknown mechanical characteristics. As a result, we determine the components of the SSS as the functions of the unknown mechanical characteristics of a homogeneous material modeling the composite. Having chosen the matching conditions, we find the mechanical characteristics of the viscoelastic transtropic composite, which are the functions of the mechanical characteristics of the matrix and fiber, as well as the volume fraction of the fiber in the composite.

Let us consider the solution to the longitudinal shear problem for a transtropic cylindrical body [1]. We give the main relations that characterize the simple longitudinal shear in the cylindrical region for a viscoelastic material (Fig. 1). Then the components of the SSSD are determined by the relations

$$\sigma_z = \sigma_r = \sigma_\theta = \sigma_{r\theta} = 0\,,\ \sigma_{zr} = \sigma_{zr}(r,\theta,t)\,,\ \sigma_{\theta z} = \sigma_{\theta z}(r,\theta,t)\,,$$

$$\varepsilon_z = \varepsilon_r = \varepsilon_\theta = \gamma_{r\theta} = 0\,,\ \gamma_{zr} = \gamma_{zr}(r,\theta,t)\,,\ \gamma_{\theta z} = \gamma_{\theta z}(r,\theta,t)\,.$$

It is assumed that the outer cylindrical surface of the region is subjected to an external load

$$\sigma_{zr}(b,\theta,t) = \sigma_0 \cos\theta\,. \tag{1}$$

In this case the axial displacement is determined by the equality

$$u_z(r,\theta,t) = \left(C_1(t) r + C_2(t)/r \right)\cos\theta\,, \tag{2}$$

where $C_1(t)$ and $C_2(t)$ are time-dependent functions. Using the Cauchy relations, we obtain the expressions for the deformations

$$\gamma_{zr} = (r,\theta,t) = \left(C_1(t) - C_2(t)/r^2 \right)\cos\theta\,,\ \gamma_{z\theta} = (r,\theta,t) = -\left(C_1(t) - C_2(t)/r^2 \right)\sin\theta\,. \tag{3}$$

Then the expressions for the stresses take the form

$$\sigma_{zr} = (r,\theta,t) = \tilde{G}_{12}\left(C_1(t) - C_2(t)/r^2 \right)\cos\theta\,,\ \sigma_{z\theta} = (r,\theta,t) = -\tilde{G}_{12}\left(C_1(t) - C_2(t)/r^2 \right)\sin\theta\,. \tag{4}$$

In the equations (4), $\tilde{G}_{12}$ is a linear integral operator characterizing the viscoelastic properties of the material

$$\tilde{G}_{12}[y(t)] = G_{12}\left(y(t) - \int_0^t R(t-\tau)\,y(\tau)\,d\tau \right), \tag{5}$$

where G_{12} is the instantaneous shear modulus corresponding to the value $t = 0$, $R(t)$ is the relaxation kernel.

2. The joint longitudinal shear of the matrix and fiber

Let us consider the problem of joint longitudinal shear of a hollow cylinder ($a \le r \le b$), modeling the matrix, and a solid cylinder ($0 \le r \le a$), modeling the fiber. The asterisk denotes the values related to the matrix, the circle refers to the fiber.

The basic relations that describe the SSS of a viscoelastic matrix on the basis of the formulas (2) – (4) can be presented as:

$$u_z^*(r,\theta,t) = \left(A(t) + B(t)/r \right)\cos\theta\,, \tag{6}$$

$$\gamma_{zr}^*(r,\theta,t) = \left(A(t) - B(t)/r^2 \right)\cos\theta\,,\ \gamma_{z\theta}^*(r,\theta,t) = -\left(A(t) + B(t)/r^2 \right)\sin\theta\,, \tag{7}$$

$$\sigma_{zr}^*(r,\theta,t) = \tilde{G}_{12}^*\left(A(t) - B(t)/r^2 \right)\cos\theta\,,\ \sigma_{z\theta}^*(r,\theta,t) = -\tilde{G}_{12}^*\left(A(t) + B(t)/r^2 \right)\sin\theta \tag{8}$$

where the linear integral operator $\tilde{G}_{12}^*$ has the form similar to that of the operator (5), with the instantaneous shift modulus G_{12}^* instead of G_{12} and the relaxation kernel $R^*(t)$ instead of $R(t)$.

The basic relations that describe the fiber SSS (solid cylinder), taking into account the limited motion at $r = 0$, take the form:

$$u_z^\circ(r,\theta,t) = C(t)\, r\cos\theta\,, \tag{9}$$

$$\gamma_{zr}^\circ(r,\theta,t) = C(t)\cos\theta\,,\ \gamma_{z\theta}^\circ(r,\theta,t) = -C(t)\sin\theta\,, \tag{10}$$

$$\sigma_{zr}^\circ(r,\theta,t) = G_{12}^\circ C(t)\cos\theta\,,\ \sigma_{z\theta}^\circ(r,\theta,t) = -G_{12}^\circ C(t)\sin\theta\,. \tag{11}$$

We find the functions $A(t)$, $B(t)$, and $C(t)$ in the relations (6) – (11) for the problem of the joint longitudinal shear of a viscoelastic matrix and elastic fiber. We use the boundary value condition (1) and the continuity conditions for the displacements and stresses in the composite at $r=a$

$$\sigma^{\circ}_{zr}(a,\theta,t)=\sigma^{*}_{zr}(a,\theta,t)\,,\ u^{\circ}_{z}(a,\theta,t)=u^{*}_{z}(a,\theta,t)\,.$$

Taking into account (5), we obtain a system of convolution-type integral equations with respect to the unknown functions $A(t)$, $B(t)$, and $C(t)$ and apply the Laplace transform to it. From the obtained system we find the following expressions for the images of the required functions:

$$\tilde{A}(p)=\frac{\left(G^{\circ}_{12}+G^{*}_{12}\left(1-\tilde{R}^{*}(p)\right)\right)\sigma_0}{G^{*}_{12}p\left(1-\tilde{R}^{*}(p)\right)\left(G^{\circ}_{12}(1+f)+G^{*}_{12}\left(1-\tilde{R}^{*}(p)\right)\right)(1-f)}\,, \tag{12}$$

$$\tilde{B}(p)=\frac{a^2\left(G^{*}_{12}\left(1-\tilde{R}^{*}(p)\right)-G^{\circ}_{12}\right)\sigma_0}{G^{*}_{12}p\left(1-\tilde{R}^{*}(p)\right)\left(G^{\circ}_{12}(1+f)+G^{*}_{12}\left(1-\tilde{R}^{*}(p)\right)\right)(1-f)}\,, \tag{13}$$

$$\tilde{C}(p)=\tilde{A}(p)+\frac{\tilde{B}(p)}{a^2}\,. \tag{14}$$

In (12) – (14) p is the Laplace transform parameter, $\tilde{R}^{*}(p)$ is the image of the relaxation kernel for the matrix material.

3. Longitudinal shear for a composite material

Let us solve a similar problem with regard to the simple longitudinal shear for a homogeneous transtropic viscoelastic material modeling the composite, representing it as a solid infinite cylinder with radius b. The boundary value condition has the form (1). The SSS components are determined by formulas similar to (9) – (11), where instead of G°_{12} there will appear the linear integral operator $\tilde{G}_{12}$ of the type (5) with the composite instantaneous shear modulus G_{12} and relaxation kernel $R(t)$. $C(t)$ is replaced by $\sigma_0\tilde{G}^{-1}_{12}$.

4. Determination of the composite shear modulus characteristics

We find the instantaneous shear modulus G_{12} and relaxation kernel $R(t)$, using the kinematic matching condition $u_z(b,\theta,t)=u^{*}_{z}(b,\theta,t)$, where $u_z(r,\theta,t)$ is the composite axial displacement. Substituting here the corresponding expressions for the axial displacements and applying the Laplace transform, we obtain the equation in the images

$$G_{12}\left(1-\tilde{R}(p)\right)=\frac{G^{*}_{12}\left(\tilde{R}^{*}(p)\right)\left(G^{\circ}_{12}(1+f)+G^{*}_{12}(1-f)\left(1-\tilde{R}^{*}(p)\right)\right)}{(1-f)G^{\circ}_{12}+(1+f)G^{*}_{12}\left(1-\tilde{R}^{*}(p)\right)}\,, \tag{15}$$

where $\tilde{R}(p)$ is the image of $R(t)$. Passing in this equality to the limit at $p\to\infty$ and taking into account that $\lim\limits_{p\to\infty}\tilde{R}(p)=\lim\limits_{p\to\infty}\tilde{R}^{*}(p)=0$, we obtain a formula for determining the instantaneous shear modulus

$$G_{12}=\frac{G^{*}_{12}\left(G^{\circ}_{12}(1+f)+G^{*}_{12}(1-f)\right)}{(1-f)G^{\circ}_{12}+(1+f)G^{*}_{12}}\,. \tag{16}$$

We note that the formula (16) coincides with the expression for the shear modulus of a composite with an elastic matrix and fiber obtained in [3].

Let us find the transformation kernel $R(t)$. For this we find its image $\tilde{R}(p)$ from the equation (15) $\tilde{R}(p)=\dfrac{C_1x^2+C_2x+C_3}{C_4(x+C_5)}$, where $C_1=(f-1)\left(G^{*}_{12}\right)^2$, $C_2=G^{*}_{12}\left(G_{12}-G^{\circ}_{12}\right)(1+f)$, $C_3=G_{12}G^{\circ}_{12}(1-f)$, $C_4=G_{12}G^{*}_{12}(1+f)$, $C_5=\dfrac{G^{\circ}_{12}(1-f)}{G^{*}_{12}(1+f)}$, $x=1-\tilde{R}^{*}(p)$.

Let the matrix viscoelastic properties be described by the integral operator with an exponential kernel of the form $R^*(t) = s_1 e^{s_0 t}$, where s_0, s_1 are the material rheological characteristics. Then the original $R(t)$ has the form

$$R(t) = q_1 e^{s_0 t} + q_2 e^{p_0 t}, \qquad (17)$$

where $p_0 = s_0 + \frac{s_1}{C_5 + 1}$, $q_1 = -\frac{C_1 s_1}{C_4}$, $q_2 = \frac{s_1(C_1 C_5^2 - C_2 C_5 + C_3)}{C_4 (C_5 + 1)^2}$.

5. Numerical results

We construct the integral operator $\tilde{G}_{12}$ for the case when the matrix is the 67L rubber with the difference relaxation kernel $R^*(t-\tau) = s_1 e^{s_0 (t-\tau)}$, where $s_0 = -1$, $s_1 = \frac{G_0^* - G_\infty^*}{G_0^*}$ with the instantaneous shear modulus G_0^*=1.5 MPa and the long shear modulus G_∞^*=0.78 MPa, $G_{12}^* = G_0^*$, the fiber material is the 23KNTS polyamide cord, for which G_{12}°=4.9 MPa. The values of the instantaneous shear modulus G_{12} for different values of the fiber volume fraction f are represented as follows:

f	0,0	0,2	0,4	0,6	0,8	1,0
G_{12}	1,500	1,857	2,310	2,904	3,717	4,900

To analyze the composite rheological properties, we construct the relationship $g(t) = \tilde{G}_{12}[1] = G_{12}\left(1 - \int_0^t R(t-\tau)d\tau\right)$.

Using (17), we obtain $g(t) = G_{12}\left(\alpha_1 + \alpha_2 e^{s_0 t} + \alpha_3 e^{p_0 t}\right)$, where $\alpha_1 = 1 + \frac{q_1}{s_0} + \frac{q_2}{p_0}$, $\alpha_2 = -\frac{q_1}{s_0}$, $\alpha_3 = -\frac{q_2}{p_0}$.

Let us investigate the viscoelastic properties of a composite for different values of f using the dimensionless function $g(t)/G_{12}$ (Fig. 2). As can be seen, the most vivid manifestation of viscoelastic properties can be observed when a composite consists entirely of the viscoelastic matrix material (at f=0). As the elastic fiber volume fraction increases, the composite viscoelastic properties manifest less and when the composite is a fiber material (at f=1), it becomes purely elastic.

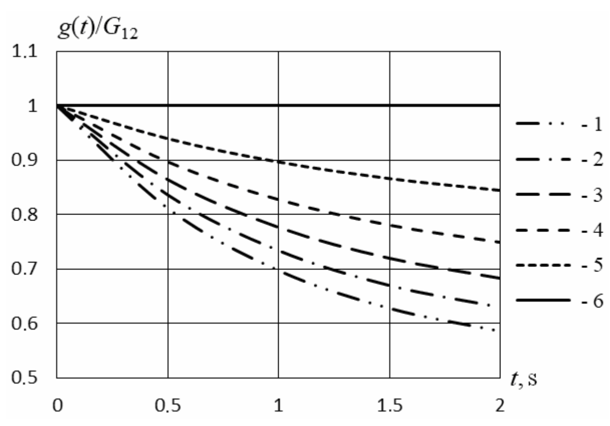

Fig. 2. Dependences g(t)/G₁₂ for different fiber content: 1 – f=0; 2 – f=0.2; 3 – f=0.4; 4 – f=0.6; 5 – f=0.8; 6 – f=1

Conclusions

The solution to the problem of finding the effective shear modulus for a composite with a viscoelastic matrix can be obtained using the kinematic matching conditions by solving two boundary value problems: with regard to the longitudinal shear of a transonic viscoelastic solid cylinder modeling the composite and the joint shear of the hollow and solid cylinders that model the matrix and fiber, respectively. The proposed technique, based on the application of the matching conditions of the selected displacement components for the cell of a homogeneous composite and its constituents, can be used to determine other effective characteristics of viscoelastic composite materials.

References

1. Klasztorny, M., Konderla, P., & Piekarski, R. (2009). An Exact Stiffness Theory of Unidirectional xFRP Composites. *Mech. Composite Materials*, vol. 45, no. 1, pp. 77–104.
2. Grebenyuk, S. N. (2012). *Opredeleniye modulya sdviga kompozitsionnogo materiala s transtropnymi matritsey i voloknom* [Determination of the shear modulus of composite material with transtropic matrix and fiber]. *Metody*

Rozviazuvannia Prykladnykh Zadach Mekhaniky Deformivnoho Tverdoho Tila: zb. nauk. prats– Methods of Solving Applied Problems of a Deformed Solid: collected scientific papers . Dnipropetrovsk: Nauka i osvita, iss. 13, pp. 92–98 [in Russian].
3. Grebenyuk, S .N. (2004). The shear modulus of a composite material with a transversely isotropic matrix and a fibre. *J. Appl. Math. and Mech.,* vol. 78, no. 2, pp. 270–276.
4. Plume, E. Z. (1992). *Sravnitelnyy analiz polzuchesti odnonapravlennykh kompozitov, armirovannykh voloknami razlichnogo tipa* [Comparative analysis of creep of unidirectional composites reinforced with fibers of various types]. *Mekhanika Kompozit. Materialov – Mechanics of Composite Materials*, no. 4, pp. 557–566 [in Russian].
5. Maksimov, R. D., & Plume, E. Z. (1984). Creep of unidirectionally reinforced polymer composites. *Mech. Composite Materials*, no. 20, pp. 149–157.
6. Kaminskiy, A. A., & Selivanov, M. F. (2005). *Ob odnom metode opredeleniya kharakteristik vyazkouprugogo deformirovaniya kompozitov* [On a method for determining the characteristics of viscoelastic deformation of composites]. *Prikl. mekhanika – International Applied Mechanics,* vol. 41, no. 5, pp. 9–21 [in Russian].
7. Boughammoura, A. (2013). Homogenization of a highly heterogeneous elastic-viscoelastic composite materials. *Mediterranean J. Math*., vol. 10, iss. 4, pp. 1793–1812.
8. Zhang, Y., Ellyin, F., & Zhang, Y. (2005). Nonlinear viscoelastic micromechanical analysis of fibre-reinforced polymer laminates with damage evolution. *Intern. J. Solids and Structures,* vol. 42, iss. 2, pp. 591–604.

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Модуль зсуву волокнистого композиту з транстропною в'язкопружною матрицею та транстропним пружним волокном

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Під час розв'язання задач механіки деформівного твердого тіла неоднорідний композиційний матеріал моделюється однорідним з осередненими механічними властивостями – ефективними характеристиками. Метою цієї статті є розробка методики визначення ефективного модуля зсуву для в'язкопружного волокнистого композита з транстропними матрицею та волокном. Їхні площини ізотропії співпадають та перпендикулярні осі волокна. Ефективний модуль зсуву визначається як функція механічних властивостей матриці та волокна і об'ємного вмісту кожного з них в композиті. Розглядається односпрямований композиційний матеріал з гексагональною схемою укладки волокон та з елементарною коміркою, що складається з в'язкопружної матриці та пружного волокна. Геометричною моделлю композита є комбінація двох коаксіальних нескінченних циліндрів – порожнистого, що моделює матрицю, та вставленого у нього суцільного, що моделює волокно. Об'єм гексагональної комірки апроксимується об'ємом циліндра. При цьому радіус циліндра обирається так, щоб об'ємний вміст волокна в гексагональній комірці співпадав зі значенням цієї характеристики для циліндричної комірки. Для опису в'язкопружних властивостей композита використовуються співвідношення спадкової теорії Больцмана-Вольтерра. Модуль зсуву визначається як інтегральний оператор з різницевим ядром. Розглянуто дві крайові задачі: про повздовжній зсув транстропного в'язкопружного суцільного циліндра, що моделює композит, та про спільний повздовжній зсув порожнистого та суцільного циліндрів, що моделюють відповідно матеріал матриці та матеріал волокна. Передбачається неперервність переміщень та дотичних напружень на поверхні контакту матриці та волокна. На зовнішній поверхні циліндричної комірки прикладається дотичне гармонічне навантаження. Для розв'язання таких задач використовується перетворення Лапласа. Як умова узгодження застосовується рівність переміщень на зовнішній поверхні циліндра для обох задач. Використання запропонованої методики дозволяє визначати характеристики інтегрального оператора, що описує модуль зсуву для в'язкопружного композиційного матеріалу. Знаходяться миттєвий модуль зсуву та параметри ядра релаксації як функції відомих механічних характеристик матриці та волокна. Як приклад визначені характеристики модуля зсуву для композиційного матеріалу, що складається з гумової матриці та поліамідного волокна.

***Ключові слова**: композит, ефективний модуль зсуву, в'язкопружність, транстропний матеріал.*

Література

1. Klasztorny M., Konderla P., Piekarski R. An Exact Stiffness Theory of Unidirectional xFRP Composites. *Mech. Composite Materials*. 2009. Т. 45. № 1. P. 77–104.

2. Гребенюк С. Н. Определение модуля сдвига композиционного материала с транстропными матрицей и волокном. *Методи розв'язування прикладних задач механіки деформівного твердого тіла*. Дніпропетровськ, 2012. Вип. 13. С. 92–98.
3. *Grebenyuk S .N.* The shear modulus of a composite material with a transversely isotropic matrix and a fibre. *J. Appl. Math. and Mech.* 2014. Vol. 78. № 2. P. 270–276.
4. Плуме Э. З. Сравнительный анализ ползучести однонаправленных композитов, армированных волокнами различного типа. *Механика композит. материалов*. 1992. № 4. С. 557–566.
5. Maksimov R. D., Plume E. Z. Creep of unidirectionally reinforced polymer composites. *Mech. Composite Materials*. 1984. № 20. P. 149–157.
6. Каминский А. А., Селиванов М. Ф. Об одном методе определения характеристик вязкоупругого деформирования композитов. *Прикл. механика*. 2005. Т. 41. № 5. С. 9–21.
7. Boughammoura A. Homogenization of a Highly Heterogeneous Elastic-Viscoelastic Composite Materials. *Mediterranean J. Math*. 2013. Vol. 10. Iss. 4. P. 1793–1812.
8. Zhang Y., Ellyin F., Zhang Y. Nonlinear viscoelastic micromechanical analysis of fibre-reinforced polymer laminates with damage evolution. *Intern. J. Solids and Structures*. 2005. Vol. 42. Iss. 2. P. 591–604.

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USE OF REFINED FINITE ELEMENT MODELS FOR SOLVING THE CONTACT THERMOALASTICITY PROBLEM OF GAS TURBINE ROTORS

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A refined mathematical model of gas turbine engine rotors using three-dimensional finite elements of a curvilinear form is developed. All the calculations were performed for rotors, which are widely used in power machine building and shipbuilding. The fact is that such components have a constructive heterogeneity that can hardly be correctly explained using well-known finite elements and their shape functions. On the other hand, the mathematical model should be as simple as possible with a view to its wide use in the process of designing a rotor. Therefore, a new refined finite-element mathematical model was developed, consisting of three-dimensional curvilinear hexahedral finite elements. It was used to calculate the displacement field caused by the complex action of the heat flux and contact load at the junction of rotor elements. This approach makes it possible to describe the entire rotor as a superposition of the developed curvilinear finite element models and make the calculation process more correct and compact. To solve this problem, a system of matrix equations was compiled. It is based on the use of energy balance dependences in the mechanical contact interaction of rotor elements, as well as the heat balance under the influence of non-stationary heat flow. When creating a numerical algorithm for solving the problem, the direct decomposition of Cholesky was used. To make the solution more compact, the Sherman scheme was used. All the calculations of displacement and temperature fields were carried out for two widely used types of joints, which are used to create such rotors, namely: joints with clearance and interference.

Keywords: *three-dimensional finite elements, gas turbine rotors, displacement and temperature fields, contact thermoelasticity problem, clearance, interference.*

Introduction

The working process of gas turbine rotors that are used in modern turbines is constantly influenced by various high intensity mechanical and thermal effects. This causes changes in the stress-deformed state of the entire motor as well as its components, such as the disk, shaft, and blades due to their mechanical contact and the heat flow passing through their contacting surfaces. This correlation is especially important for gas turbine engine components due to their extremely complex working process.

It should be noticed that the main conditions of contact between rotor components are not always identical even when one-type parts contact. [1]. Firstly, the shaft and rotor are mounted before the start of the working process. This means that each pair of contacting surfaces has its own definite conjugation conditions. But during the working process the conjugation conditions can rapidly change. This fact causes the changes of the mechanical contact pressure. Therefore, changes of heat flow parameters can also be observed on the shaft and blade row contact surfaces [2]. Therefore, the mathematical model used for solving a gas turbine engine rotor thermoelastic-