SCREW-TYPE SYMMETRY IN MACHINE COMPONENTS AND DESIGN AT IMPLEMENTATION ON A 3D PRINTER

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The creation of mathematical models for the implementation of 3D printing is of considerable interest, which is associated with the active introduction of 3D printing in various industries. The advantages of using 3D printing are the following: the possibility of making non-standard models, reducing the time to create new prototypes, making simple and low cost products, and using modern super-strength materials. The manufacture of machine components with a screw-type symmetry occurs in various, often very complicated ways to include casting with the subsequent rotary machining, methods of hot deformation, electrophysical and electrochemical methods, etc. Their manufacture on a 3D printer can be very promising. In this paper, the R-functions theory is used for the mathematical and computer modeling of geometric objects with a screw-type symmetry for the case when 3D printing technology is to be implemented. An analytical recording of the objects being designed makes it possible to use alphabetic geometric parameters, complex superposition of functions, which, in turn, allows us to quickly change their structural elements. The working part of many mechanisms for moving a material along a helical rotating surface is a screw. Screws are used instead of wheels in some types of cross-country vehicles or combines. They are an indispensable part in extruders and boring stations. At large enterprises, they are used as a means of transporting bulk materials. Screws are indispensable in the food industry. Among other things, they are used in small arms, where they play the role magazines for cartridges. In this work, we build mathematical and computer models of the variable and constant pitch screws to be implemented on a 3D printer. In order to organize and intensify various processes in power plants and other technical devices, swirling is widely used. Swirling is an effective means of stabilizing the flame in the combustion chambers of gas turbine engines. It is also used to intensify heat and mass transfer in channels; in chemical, petroleum, gas and other industries. This paper considers mathematical and computer models of a screw swirler, locally twisted tube, and complex cross-section twisted tube intended for the implementation on a 3D printer. The process of building a desk lamp with a design in the form of elliptic twisted tori is shown as well.

Keywords: R-functions theory, 3D printing, screw-type symmetry, screw.

Introduction
Currently, the creation of volumetric models of any items is accompanied by the growth in the popularity of using 3D printers. The technology of 3D printing is based on the principle of creating a layerwise solid model. The advantages of such devices over those using the conventional methods of creating models are high speed, simplicity, and low cost.

There are different approaches to creating 3D models: 3D scanning of an existing object, obtaining a 3D model using a fairly wide range of software products whose libraries contain primitive forms of various types, and creating a 3D model using the mathematical apparatus of analytical geometry. In cases where the object to be simulated exists physically, the most convenient way to obtain a 3D model is to use a 3D scanner. However, when the object to be simulated has large dimensions or does not exist at all, such an approach is impossible. In such cases, the creation of 3D-models can be implemented using software products. But this method has a number of disadvantages too, for example, a limited set of primitives in the library modules of the programs. Here we should especially note the geometric objects with a screw-type symmetry in whose cross-sections there are no classical regions, but those composed of the known primitives [1]. In these cases, the creation of a mathematical model of the geometric object with the help of the apparatus of analytical geometry is very promising in the field of 3D modeling. For the construction of the equations of geometric objects in an analytical form, the R-functions theory can prove to be very useful. It gives the possibility of creating complex geometric objects without approximations. In this paper, we consider the application of the R-functions theory for the creation of mathematical and computer models of geometric objects with screw-type symmetry for the implementation on a 3D printer.
superpositions and introducing literal parameters into the logical formula. These parameters allow us to quickly change the shape of the object being designed [2–4]. Also convenient is the fact that the process of creating a model occurs in stages and there is an opportunity to introduce changes to the structure being designed.

Thus, the expansion of the field of application of the apparatus of the R-functions theory for creating the mathematical models of the engineering objects to be built on a 3D printer is an urgent scientific and technical problem.

The aim of this work is to create mathematical and computer models of geometric objects with a screw-type symmetry on the basis of the R-functions theory in order to implement 3D printing technology.

**Main Part**

R-operations [2] $fk \wedge_0 \fl = fk + \fl - \sqrt{fk^2 + \fl^2}$; $fk \lor_0 \fl = fk + \fl + \sqrt{fk^2 + \fl^2}$ and a helical non-orthogonal coordinate system [4] \begin{equation*} \begin{cases} x_1 = x \cos \alpha + y \sin \alpha \\ y_1 = -x \sin \alpha + y \cos \alpha \end{cases} \end{equation*} are used in this work. When constructing the equations corresponding to geometric objects with cyclic-type point symmetry, to reduce the number of R-operations, we will use the results of the following theorem [4].

**Theorem.** Let the translational region $\Sigma_0 = [\sigma_0(x, y, z) \geq 0]$ be symmetric about the abscissa axis, and the region $\Sigma_1 = [\sigma_1(x - r_0, y, z) \geq 0]$ can be located inside the sector $-\alpha \leq \theta \leq \alpha$, $0 < \alpha < \frac{\pi}{n}$. The regions $\Sigma_k = [\sigma_k(r \cos(\theta - 2\pi k/n) - r_0, r \sin(\theta - 2\pi k/n), z) \geq 0]$ have been obtained by rotating the region $\Sigma_1$ in the $xOy$ plane around the origin of the coordinates to the angles $2\pi k/n$. Then the equation of the boundary $\partial \Omega$ of the region $\Omega = \bigcup_{k=0}^{n-1} \Sigma_k$ has the form

$$\omega(x, y) = \sigma_0(r \cos \mu(\theta, n) - r_0, r \sin \mu(\theta, n), z) = 0,$$

where $r = \sqrt{x^2 + y^2}$, $\theta = \arctg \frac{y}{x}$,

$$\mu(\theta) = \frac{8}{n\pi} \sum_{k=1}^{\infty} (-1)^{k+1} \frac{\sin \left[ \frac{(2k-1)n\theta}{2} \right]}{(2k-1)^2}.$$ 

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In the process of numerical implementation, for the construction of the equation of the boundary $\partial \Omega$ with the rational accuracy, the number of the series members $k = 5$ is sufficient.

In addition, to construct an object of a given thickness $\delta$, its normalized equation $\omega(x, y, z) \geq 0, \frac{\partial \omega}{\partial n}|_{\partial \Omega} = 1$ was used. It is well known that the function $\omega(x, y, z)$ in the neighborhood of the

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Fig. 1. Screw: a – $N$ – variable pitch; b – $N = 5$; c – $N = 3$
boundary $\partial\Omega$ behaves like the distance to $\partial\Omega$. Then the function $\omega_l(x,y,z) = \frac{\delta}{2} - |\omega(x,y,z)| \geq 0$ will be
positive only in the belt width $\delta$. In some cases, the use of this approach can significantly reduce the number of R-operations, and consequently, the computational process.

The working part of many mechanisms for moving a material along a helical rotating surface is a screw. The screw is a metal rod with a helical surface. The field of their application is rather wide. They are used instead of wheels in some types of cross-country vehicles or combines. They are indispensable components in extruders and at boring stations. At large enterprises, screws are used as a means of transporting bulk materials. Among other things, screws are used in small arms, where they play the role of magazines for cartridges. Screw production occurs in various, often very complex ways, to include casting with the subsequent rotary machining, hot deformation and cold bending, welding, etc. The 3D printing of screws can prove to be very promising. To make the latter possible, we construct a mathematical and computer model for both a variable and constant pitch screw (Fig. 1).

![Fig. 1. Screw swirler:](image)

In this case, of particular interest is the construction of a variable twist pitch in the form of a continuous function $N = f(z)$.

In power plants and other technical devices, to organize and intensify various processes, flow swirling is widely used.

Flow swirling is an effective means of stabilizing the flame in the combustion chambers of gas turbine engines; it is used to intensify heat and mass transfer in channels in the chemical, oil, gas and other industries [1]. The swirlers used in practice differ in the method and nature of swirl and in the length of the swirling device. Consider the screw swirler (Fig. 2), locally twisted tube (Fig. 3), and complex cross-section twisted tube (Fig. 4) that were proposed in [1].

**Screw Swirler**

\[
\rho = \sqrt{x^2 + y^2}; \theta = \arctg \frac{y}{x}; zz = z - \frac{\theta h}{\pi} z; ff = \frac{\pi}{h} zz;
\]

\[
\mu = 4h \sum_{k} (-1)^{k+1} \sin\left(\frac{(2k-1)ff}{(2k-1)^2}\right);
\]

\[
fp1 = 0.04 - \frac{(\rho - 0.8)^2 - \mu^2}{0.4} \geq 0;
\]

\[
fp2 = \frac{1 - x^2 - y^2}{2} \geq 0; \quad fp3 = \frac{(0.36 - x^2 - y^2)}{1.2} \geq 0;
\]

\[
W = fp1 \vee fp3 \geq 0 .
\]

At $h=1$ (Fig. 2, a); at $h=2$ (Fig. 2, b).
Local Twist Tube

\[ a = 1; b = 5; l = 1; f_{i1} = 2\pi \frac{z}{h}; f_{i2} = 2\pi \frac{l}{h}; f_{i3} = 2\pi \frac{z-b+a}{h}; \]
\[ f_{i12} = (f_{i1} + f_{i2} - \text{abs}(f_{i1} - f_{i2}))/2; \]
\[ \alpha = (f_{i12} + f_{i3} + \text{abs}(f_{i12} - f_{i3}))/2; \]
\[ \begin{align*}
xx &= x \cos \alpha + y \sin \alpha \\
yy &= -x \sin \alpha + y \cos \alpha \\
\end{align*} \]
\[ \omega_1 = 1 - xx^2 - \frac{yy^2}{1.5^2} \geq 0; \]
\[ W = \alpha \wedge_0 (10-z)(z+5) \geq 0 . \]

At \( h = 7 \) (Fig. 3, a); at \( h = 5 \) (Fig. 3, b).

It should be noted that specifying information in an analytical form with the use of literal parameters allows us to quickly introduce changes to the form of the object under study.

Complex Cross-Section Tube

\[ \alpha = 0; (\alpha = \frac{\pi z}{40}; \alpha = \frac{\pi z}{30}; \alpha = \frac{\pi z}{20}); \]
\[ \begin{align*}
xx &= x \cos \alpha + y \sin \alpha; \\
yy &= -x \sin \alpha + y \cos \alpha; \\
f_{kv} &= \frac{10^2 - xx^2}{20} \wedge_0 \frac{10^2 - yy^2}{20} \geq 0; \\
f_{91} &= \frac{16 - xx^2}{8} \geq 0; \\
f_{92} &= \frac{16 - yy^2}{8} \geq 0; \\
f_{r} &= \frac{ro^2 - f_{91}^2 - f_{92}^2}{2} \geq 0; \\
f_{9} &= f_{91} + f_{92} + \left( f_{91}^2 + f_{92}^2 + \frac{fr}{8ro^2} \left| fr \right| + \left| fr \right| \right) \geq 0; \\
f_{11} &= f_{9} \wedge_0 f_{kv} \geq 0; \\
f_{11o} &= \left( \frac{4^2 - (xx-10)^2 - yy^2}{8} \right) \vee_0 \left( \frac{4^2 - (xx+10)^2 - yy^2}{8} \right) \geq 0; \\
f_{12o} &= \left( \frac{4^2 - (yy-10)^2 - xx^2}{8} \right) \vee_0 \left( \frac{4^2 - (yy+10)^2 - xx^2}{8} \right) \geq 0; \\
f_{lo} &= f_{11o} \vee_0 f_{12o} \geq 0; \\
f_{lk} &= f_{11t} \vee_0 f_{lo} \geq 0; \\
f_{lk} &= h-t \left| f_{1k} \right| \geq 0; \\
\end{align*} \]
\[ W = f_{1} \wedge_0 \left( L^2 - z^2 \right) \geq 0 \text{ (Fig. 4).} \]

It should be noted that the tube of the cross-section shown is very difficult to manufacture, especially if we need to change the twist parameter. Therefore, it is advisable to do it on a 3D printer. Since, by changing the parameter \( \alpha \), we obtain a required twist pitch tube, ready to be 3D printed, then by changing the parameters \( ht \) and \( L \), we obtain a pipe of the desired thickness and length.

Consider the process of building a designer desk lamp (Fig. 5, 6). The lamp consists of a stand with holes for electrical wiring and a lampshade.

Stand

\[ p = 10; xp = x/p; yp = y/p; zp = z/p; f_{1} = \frac{P}{19} \left( 9.5^2 - xp^2 - yp^2 - \left( zp + 8 \right)^2 \right) \geq 0; \]
Shade

\[ f_5 = \frac{2 - \left( \frac{9}{4} \right) \left( x^2 + y^2 - (z - 6.5)^2 \right)}{0.25} \geq 0; \]

\[ f_6 = \frac{2 - \left( \frac{9}{4} \right) \left( z - 6.5 \right)^2}{0.25} \geq 0; \]

\[ Ra = 4.8; Na = 10; \alpha = \arctg \frac{y_p}{x_p}; X_1 = \sqrt{x_p^2 + y_p^2} - Ra; za = z - 7.3; \]

\[
\begin{cases}
XXa = X_1a \cos \alpha + za \sin \alpha \\
ZZa = -X_1a \sin \alpha + za \cos \alpha
\end{cases}
\]

\[
\begin{cases}
XXa = X_1a \cos \alpha + za \sin \alpha \\
ZZa = -X_1a \sin \alpha + za \cos \alpha
\end{cases}
\]

\[ tora = t_1a \lor t_2a \geq 0; \]

\[ \omega_a = \omega_a \lor \omega_{tor} \geq 0; \]

\[ fd = \left( 0.5^2 - x^2 \right) \lor \left( 0.5^2 - y^2 \right) \geq 0; \]

\[ fbl = \left( 0.1 - (7 - zp) \right) \lor \left( \frac{p}{9} \left( 4.5^2 - x^2 - y^2 + (z - 6.5)^2 \right) \right) \geq 0; \]

\[ \omega_{per} = \left( \omega_{a} \lor \omega_{bl} \right) \lor \left( x^2 + y^2 - 0.5^2 \right) \geq 0; \]

\[ \omega_{abaj} = \omega_a \lor \omega_{per} \geq 0; \]

\[ \omega_{lampa} = \omega_{abaj} \lor \omega_{podst} \geq 0 \]

By changing the values of the letter parameters \( R, Ra, zn, za \) and setting \( t_2 = 0, t_2a = 0 \) we obtain the lamp design with the pattern shown in Fig. 7.

Thus, any sizes characterizing this object can be changed.

The visualization of all the constructed models was carried out using the RFPreview program [5, 6].

Fig. 8 shows all the 3D printed models.

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Fig. 5. Step-by-step lamp base construction:
- a − lower part;
- b − column added;
- c − holes and wire grooves added.

Fig. 6. Step-by-step construction of the lampshade and whole lamp:
- a − lampshade;
- b − cross-shaped mount added;
- c − lamp assembly.
The constructed mathematical models have been implemented on an Anet A8 3D printer using the most inexpensive and affordable Fused Deposition Modeling (FDM) technology. Its main disadvantages are the capability of using only plastic, the sensitivity to temperature drops, plastic spreading and need to organize supporting structures. The use of other more advanced 3D printing technologies (for example, Selective Laser Sintering (SLS), in which nylon, glass, plastic, ceramics, and various powdered metals can be used as raw materials) will improve the appearance of the printed products and their strength characteristics.

Conclusions

The creation of mathematical models for the implementation of 3D printing is of considerable interest, which is associated with the active introduction of 3D printing in various industries. The advantages of using 3D printing are obvious. These are the production of non-standard models, reduction of time for the creation of new prototypes, reduction of the repair time, simplicity and considerable cheapness of production, and use of modern super-strength materials. In this paper, we developed a general approach to the construction of mathematical and computer models of geometric helical symmetry objects, with the models based on the R-functions theory. By changing the swirl law, we change only the form of the function $\alpha$ in the corresponding coordinate transform. The reliability of the results obtained, their adequacy to the objects being designed is confirmed by visualization, both under the operating conditions of the RFPreview program, and the implementation on a 3D printer.

An analytical recording of the objects being designed makes it possible to use alphabetic geometric parameters and a complex superposition of functions, which, in turn, allows us to quickly change their structural elements. The property of positiveness of the constructed functions in the internal points of the object is very convenient for the implementation of 3D printing.

References

Гвинтовий тип симетрії в деталях машин та дизайні при реалізації на 3D-принтері

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Створення математичних моделей для реалізації 3D-друку становить значний інтерес, що пов'язаний з активним впровадженням 3D-друку в різні галузі промисловості. Переваги застосування 3D-друку: виготовлення нестандартних моделей, скорочення часу на створення нових прототипів, простота і значна дешевизна виробництва, використання сучасних надійних матеріалів. Виготовлення деталей машин з гвинтовим типом симетрії відбувається різними, часто дуже складними способами. Це літньо з подальшою праксе або обробкою, способи гарячої деформації, электрорізниці та електрохімічні способи того, що дуже перспективним може виявитися їх виготовлення на 3D-принтері. У цій роботі застосовується теорія R-функцій для математичного та комп’ютерного моделювання геометричних об’єктів з гвинтовим типом симетрії під час реалізації технології 3D-друку. Аналітичний запис проектованих об’єктів дає можливість використовувати буквені геометричні параметри, складні суперпозиції функцій, що, в свою чергу, дозволяє оперативно змінювати їхні конструктивні елементи. Робочої деталі багатьох механізмів для просування матеріалу узводж гвинтової поверхні, яка обертается, є шнек. Шнеки використовуються в плоскі високих робіт для виготовлення деталей в екструдерах і на бурових станціях. На великих підприємствах їх використовують як засіб транспортування сипучих речовин.

Ключові слова: теорія R-функцій, 3D-друк, гвинтовий тип симетрії, шнек.

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