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## CALCULATION OF INDICATORS OF RELIABILITY OF TECHNICAL SYSTEMS BY THE TYPICAL STRUCTURAL SCHEME METHOD

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A. Podgorny Institute of Mechanical Engineering Problems of NASU, 2/10, Pozharsky str., Kharkiv, 61046, Ukraine A method for calculating the indicators of structural reliability of systems with a large number of elements is presented. The method is based on the use of typical structural schemes, reflecting the concept of connections between elements. It is shown how, by supplementing and combining typical structures, one can create graphological structures to perform calculations of reliability indicators. The approach can be used in the development of algorithms and software solutions on computer problems, based on assessments of the structural reliability of systems. Such tasks, in particular, include: assessing the safety of nuclear units, planning their repairs, assessing the reliability of directional systems for transporting media, and estimating the residual resources of technical facilities. Various private methods have been developed for their solution. However, it is not possible to standardize calculations of reliability indicators because of the diversity of systems and conditions of their operation. The presented approach is focused on the automation of calculations of indicators of structural reliability of a wide class of technical systems. It is based on the proof of the existence of a calculation algorithm on a set of typical structural schemes. It is assumed that the computer recognizes images of typical structures as part of graphological images of systems. The content of the problem is as follows. A technical system is given. It is required to build a graphological image and calculate the index of its structural reliability. The proposed calculation method is based on the representation of the graphological image of the system in the form of a composition of graphological images of typical structures, the reliability indices of which are calculable. They are substituted by individual elements with calculated values of the reliability index. Such substitutions make it possible to simplify the initial graphological image of the system by reducing the total number of elements and calculate the system reliability indicator. The calculation and substitution procedure continues until the graphical image of the system has one typical structure for which we calculate the reliability index. The number of elements in the system is unlimited, since the subsitution procedure is carried out sequentially until the formation of one typical structure. A significant limitation in the application of the method to the calculation of the structural reliability of a wide range of complex technical systems is due to the limitations of many typical structures. However, such a bank of typical structures can be created and used in the development of appropriate design programs.

Keywords: algorithm, system, structural reliability, typical schemes.

## Introduction

Calculations of the reliability of systems according to the reliability of the elements included in the systems are referred to as the calculations of their structural reliability. They can be performed using the various methods described in [1-6].

The existing basic methods for assessing the structural reliability of systems include the following ones: a logical-probabilistic method, a discrete Markov process method, and a simulation modeling method. They are complex, and their use with a large number of elements causes considerable difficulties. In this regard, of interest are other methods that facilitate the solution of calculation problems.

Due to the diversity and scale of technical systems, there is no and, apparently, there will not be any universal approach to the calculation of their structural reliability.

One of the approaches used in engineering hand calculations to assess the structural reliability of a large system is shown in [7]. It consists in enlarging the elements of the structural scheme, the graphological image (GI) of the system. When there are a lot of elements in the system, the calculations become lengthy. This process becomes even more laborious if the calculations need to be made repeatedly with changing data.

This article describes a calculation method, partially developed in [8, 9], that uses the GI as a logical model of the system failure condition.

In the future, to reduce notes, the value of the reliability index (RI) of the system will also be assigned to the system GI.

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## ПРИКЛАДНА МАТЕМАТИКА

The task, which is considered in the article, is as follows. A technical system is given. It is required to build the system GI and calculate the RI of its structure.

The proposed calculation method is based on the representation of the system GI in the form of the GI composition of special parts, which are substituted by separate elements with the calculated RI values. The substitutions make it possible to simplify the initial system GI by reducing the total number of elements and calculate its RI.

#### Main part

Let X be a system. The calculation of its structural reliability is based on its GI. However, the creation of the system GI is a non-formalized process. Therefore, two independent researchers can create in some sense permissible, but different GIs of the same system. Since both the number of system elements and number of researchers are finite, then we can assume that for X there is a finite set of valid GI versions  $S_v$ ,  $v = 1,2,...,\Psi$  of this system. The values of the graphological image reliability index (GIRI)  $S_v$ ,  $v = 1,2,...,\Psi$  must be the same, as in the acceptable models of the failure condition of the same system X

$$Q(S_{\nu}) = Q_{S}, \quad \nu = 1, 2, ..., \Psi,$$
 (1)

where S is any one GI from the set  $S_v$ ,  $v = 1, 2, ..., \Psi$ , and  $Q_s$  is the value of its RI.

In terms of (1), the GI versions  $S_v$ ,  $v = 1, 2, ..., \Psi$  are equivalent, and let the GI S be their representative. To calculate the GIRI S, introduced are simple elements SE, generalized elements G, virtual connectors, as well as typical basic GIs (BGIs)  $g_0, g_1, g_2, ..., g_M$ .

Assumption 1. Taken as simple elements *SE* are such that, in the GIs under consideration, do not allow any division into components. These simple elements have either their RI values or calculation procedures determined.

Examples of simple elements SE within the system GI can be such ones as a pump or pump bearing, depending on the degree of the GI detailing.

Also, the *SE*-type elements include virtual connectors, which are intended for the organization of the GI scheme. The RI of connectors are assigned values 0 or 1.

Assumption 2. Taken as generalized elements, denoted by G, are such that, in the GIs under consideration, are formed by a set of SE-type elements and a set of BGIs.

Assumption 3. The BGIs  $g_0, g_1, g_2, ..., g_M$  are graphological structures containing the minimum number of simple elements that define the principle scheme of connections between them. BGIs have their RI calculation procedures determined, for example, of failure probability (FP) or failure-free operation probability (FFOP) within the time [0, t].

GBIs can be represented on a piece of paper, on a computer screen, or other data carrier. With their help, all the GI versions  $S_v$ ,  $v = 1, 2, ..., \Psi$  can be created.

Considered as BGIs can be:  $g_0$  – the GI displaying a single element;  $g_1 = (e_1 \lor e_2)$  – the GI showing an in-series connection of two elements;  $g_2 = (e_1 \land e_2)$  – the GI displaying a parallel connection of two elements;  $g_3$  – the GI displaying a structure that has a time reserve for restoration; redundant systems with ideal and non-ideal switches, and other GIs.

Assumption 4. The basic elements of the type SE, generalized elements G, virtual connectors, and the BGIs  $g_0, g_1, g_2, ..., g_M$  are used to create system GIs. In so doing, each element must have only one entry into the system GI and any two elements must have only one connection with each other. A set of system GI elements has a determined set of finite elements that complete the GI.

Consider the calculation of the basic graphological image reliability index (BGIRI)  $g_1 = (e_1 \lor e_2)$ . As the RI *R* of the GI  $g_1$ , we will choose the FFOP within the time [0,t]. Then, according to the well-known formula [1],  $R(g_1) = r(e_1) \cdot r(e_2)$ , where r – is the FFOP of the elements  $e_1, e_2$  within the time [0,t].

If we add another element, say,  $e_3$ , in series to  $g_1$  then, let us assume that there is no newly formed GI  $(e_1 \lor e_2 \lor e_3)$  in the BGI list  $g_0, g_1, g_2, ..., g_M$ . Then, according to assumption 3, there is no procedure for

calculating the value of the RI R of the structure  $(e_1 \lor e_2 \lor e_3)$ . The situation can be changed by declaring  $g_1$  as a BGI element. Then we obtain a generalized element (generalized GI) from the in-series connection of two elements:  $G_1 = (g_1 \lor e_3) = ((e_1 \lor e_2) \lor e_3)$ . And for such a scheme, applicable is the RI calculation formula  $R(G_1) = r(g_1) \cdot r(e_3) = r(e_1 \lor e_2) \cdot r(e_3) = r(e_1) \cdot r(e_2) \cdot r(e_3)$ .

Since the BGIs  $g_0, g_1, g_2, ..., g_M$  have the existence of RI calculation algorithms postulated, then taken as the elements  $g_1$  and  $e_3$  from the previous formula can be any BGI from  $g_0, g_1, g_2, ..., g_M$ . Then the generalized element will get the form  $G_1 = (g_k \lor g_m)$  and  $R(G_1) = r(g_k) \cdot r(g_m)$ , where  $r(g_k), r(g_m)$  is the FFOP of the BGI  $g_k, g_m$  within the time [0, t].

The structure  $G_1 = (g_k \lor g_m)$  can be supplemented with other BGIs, for example, like this:  $G_1 = ((g_k \lor g_i) \lor (g_m \lor g_j))$  where  $g_k, g_i, g_m, g_j$  is the BGI from  $g_0, g_1, g_2, ..., g_M$ , i.e. the generalized GI  $G_1$  is the GI  $g_1$  with the generalized elements  $(g_k \lor g_i), (g_m \lor g_j)$ .

However, the generalized elements  $(g_k \vee g_i), (g_m \vee g_j)$  may be absent in the BGI list  $g_0, g_1, g_2, ..., g_M$ . Consequently, according to assumption 3, the RI calculation procedure is not determined for them. To overcome this difficulty, it is necessary to carry out the appropriate calculations, namely  $R(G_1) = r(g_k \vee g_i) \cdot r(g_m \vee g_j) = r(g_k) \cdot r(g_i) \cdot r(g_m) \cdot r(g_j)$ . Spreading this technique to a more general case, we obtain the RI calculation formula for the generalized element  $G_1$  formed by the in-series connections of BGIs

$$R(G_1) = r[g_k \lor (\cdot) \lor \dots \lor (\cdot) \lor g_i(\cdot) \lor \dots \lor (\cdot)] \lor r[g_m \lor (\cdot) \lor \dots \lor (\cdot) \lor g_i(\cdot) \lor \dots \lor (\cdot)].$$

$$(2)$$

Since formula (2) contains the BGIs as components, each of them can be supplemented with other BGIs. This procedure can be repeated many times: it is unlimited. For example, suppose that the BGI  $g_1$  is a component in (2). Supplement  $g_1$  with the BGI  $g_2$ . We get  $G_{1,2} = g_1 \lor g_2 = (e_1 \lor e_2) \lor (E_1 \land E_2)$ . The supplementation of formula (2) with the generalized element  $G_{1,2}$  preserves the calculability of the RI  $G_1$ .

Similar reasoning, as above, leads to the formula for calculating the FP, within the time [0,t], of the generalized element  $G_2$  reflecting the parallel connections of BGIs

$$Q(G_2) = q[(g_k \land (\cdot) \land \dots \land (\cdot) \land g_i(\cdot) \land \dots \land (\cdot)] \land q[g_m \land (\cdot) \land \dots \land (\cdot) \land g_j(\cdot) \land \dots \land (\cdot)],$$
(3)

where q is the FP of the arguments within the time [0,t] in formula (3).

If the GI S contains the generalized GIs  $G_1 \vee G_2$  or  $G_1 \wedge G_2$ , as well as their combinations, joined by signs of disjunction  $\vee$  and conjunction  $\wedge$ , then we calculate their RI as the RI of series-parallel structures.

Consider the calculation of the BGIRI  $g_3$ . We represent it in the form of the formula:  $g_3 = g_3((A_3^1 \lor E_3 \lor A_3^2) \land A_3^3)$ , where  $E_3$  is the working component represented by a simple element *SE* in the GI of the system X ; the components;  $A_3^1, A_3^2, A_3^3$  are simple elements of the GI of the system X that provide the operation of  $E_3$ . For  $g_3$ , in assumption 1, postulated is the existence of an algorithm for calculating its RI.

However, the BGI  $g_3$  within the GI of the system X can be supplemented with switches (P), filters (F), diagnostic devices (D), automation equipment (U). Then, the BSI  $g_3$  will lose its BSI status and become some generalized structure  $G_3$ , which we will define as the dependence

$$G_{3} = G_{3}((A_{3}^{1} \lor P \lor U \lor E_{3} \lor F \lor D \lor A_{3}^{2}) \land A_{3}^{3}).$$
(4)

Accept that for GI (4), its RI calculation algorithm is not provided. However, due to the simplicity of the connections (a series-parallel connection) between the elements, GI (4) obviously reduces to the BGI  $g_3 = g_3((A_3^1 \vee E_3 \vee A_3^2) \wedge A_3^3)$ , and its RI, according to Assumption 3, is executable.

### ПРИКЛАДНА МАТЕМАТИКА

Suppose that the working component  $E_3$  of the BGI  $g_3$  is the GI, for example, consisting of the GI subsystems  $E_{3,1}, E_{3,2}, ..., E_{3,m}$ , which, in turn, consist of the GI subsystems  $E_{3,1,1}, ..., E_{3,2,1}, ..., E_{3,m,1}, ..., E_{3,m,m}$ . Representing this for clarity in the chain of inclusions, we get

$$E_{3} \supset [E_{3,1} \supset (E_{3,1,1}, E_{3,1,2}, \dots)], [E_{3,2} \supset (E_{3,2,1}, E_{3,2,2}, \dots)], \dots, [E_{3,m} \supset (E_{3,m,1}, E_{3,m,2}, \dots, E_{3,m,m})].$$
(5)

Assume that the subsystems of the last level of inclusion  $E_{3,1,1},...,E_{3,2,1},...,E_{3,m,1},...,E_{3,m,m}$  consist of BGIs with simple elements and, accordingly, calculate their RI. Then the GIs  $E_{3,1}, E_{3,2},...,E_{3,m}$  are formed from simple elements and, if they consist of BGIs, then we obtain the RI calculability  $E_{3,1}, E_{3,2},..., E_{3,m}$ , and then, respectively, the RI of the working component  $E_3$ .

Similarly to  $g_3$ , the BGI list  $g_4, g_5, ..., g_M$  can also be supplemented with the GIs of subsystems with different levels of mutual inclusion. At the same time, the BGIs  $g_4, g_5, ..., g_M$  will lose their basic status. Then they will be used to form the generalized elements  $G_4, G_5, ..., G_M$ . If the RIs of the included GIs of subsystems are calculable, then this will lead to the BGIs  $g_4, g_5, ..., g_M$  with simple elements and the calculability of their RIs.

Thus, the supplementation of the BGIs  $g_0, g_1, g_2, ..., g_M$  with new elements or generalized elements with decomposition into GI subsystems having calculable RIs will lead to new structures with calculable RIs.

The problems of calculating the structural reliability of large systems are discussed in other publications. In particular, the calculation by the method of probabilistic equivalentization is described in article [10], and that by the Monte Carlo method, in article [11]. There are also other publications where the problem of calculating the structural reliability of systems is considered from different points of view and under various conditions of a restrictive nature. However, the problem of automation of calculations is not given due attention. In this regard, the following theorem will be useful.

**Theorem.** Let there be given a system X and its GI S. If the GI S is composed of the BGIs  $g_0, g_1, g_2, ..., g_M$  with simple elements or of generalized GIs  $G_1, G_2, G_3, ..., G_M$  with calculable RIs, then the RI of the system X is calculable.

**Evidence.** Let the GIRI  $Q_{s}^{*}(t)$  be the FP within the time (0,t).

If the GI S consists of a single BGI  $g_0, g_1, g_2, ..., g_M$  composed of simple elements of the SE -type, then, according to assumption 1, the GIRI S is calculable and equal to  $Q_s^*(t)$  within the time (0,t).

Let the GI S be formed from several BGIs and there be a BGI  $g_1$  in it (the in-series connection of two elements). Calculate its RI. Let it be the FP  $q_1^{(1)}(t)$  within the time (0,t). In the GI S, we substitute the BGI  $g_1$  by the BGI  $g_0$  with the RI  $q_1^{(1)}(t)$ . Then, instead of the GI S, we will get a new GO  $S_1^{(1)}$ . In this case, the GIRI S will retain its former FP value of  $(Q(S) = Q(S_1^{(1)}) = Q_S^*(t))$  within the time (0,t).

The GI  $S_1^{(1)}$  can have several BGIs  $g_1$ . We find the BGI  $g_1$  within the GI  $S_1^{(1)}$ , and calculate its RI. Suppose, it will be  $q_1^{(2)}(t)$ . In  $S_1^{(1)}$  substitute the GI  $g_1$ , by the GI  $g_0$  with the RI  $q_1^{(2)}(t)$ . We will get a new GI  $S_1^{(1)}$  instead of the GI  $S_1^{(1)}$ , for which  $Q(S_1^{(2)}) = Q_s^*(t)$  within the time (0,t). We will repeat this procedure until, the next GI  $S_1^{(k)}$ , suppose it will be the GI  $S_1^{(k)}$ , contains no BGI  $g_1$ .

Suppose that the GI  $S_1^{(k)}$  has  $g_2$ -type BGIs with simple elements. Calculate its RI, suppose it will be  $q_2^{(1)}(t)$ . In the GI  $S_1^{(k)}$  substitute the BGI  $g_2$  by the BGI  $g_0$  with the RI  $q_2^{(1)}(t)$ . After substitution, instead of the GI  $S_1^{(k)}$ , we will receive a new GI, which we will denote by GO  $S_2^{(1)}$ , and for which the FP  $Q(S_2^{(1)}) = Q_S^{(k)}(t)$  within the time (0,t).

We will repeat these procedures until the transformed GI *S* contains the BGIs  $g_1$  and  $g_2$ . We will denote the transformed GO *S* by  $S_2^{(k)}$ , for which the RI  $Q(S_2^{(k)}) = Q_S^*(t)$  within time (0,t).

Let the GI  $S_2^{(k)}$  contain the BGI  $g_3$ . According to assumption 3, the BGIRI  $g_3$  is calculable. We assume that it is equal to the FP  $q_3^{(1)}(t)$  within the time (0,t). In the GI  $S_2^{(k)}$ , substitute the BGI  $g_3$  by the BGI  $g_0$  with the RI  $q_3^{(1)}(t)$ . After substitution, instead of the GI  $S_2^{(k)}$  we will receive a new GI, which we will denote by GO  $S_3^{(1)}$ , for which the FP  $Q(S_3^{(1)}) = Q_s^*(t)$  within the time (0,t). If the GI  $S_3^{(1)}$  contains more of the BGI  $g_3$ , then, continuing this process of its exclusion, we will receive the GI  $S_3^{(2)}$ ,  $S_3^{(3)}$ , ...,  $S_3^{(k)}$ , whose FP  $Q(S_3^{(2)}) = Q(S_3^{(3)}) = ... = Q(S_3^{(k)}) = Q_s^*(t)$  within the time (0,t).

Modifications of the GI  $S_3^{(1)}$ ,  $S_3^{(2)}$ ,..., $S_3^{(k)}$  could lead to the appearance of BGIs  $g_1$  and / or  $g_2$ , as well as, possibly, other BGIs. Then, as above, repeating the actions with these BGIs, we will obtain the BGI, suppose it will be  $S_M^{(k)}$ , which will contain only one BGI  $g_0$  with the calculated value of the GIRI S: the FP  $Q(S_M^{(k)}(t)) = Q_S^*(t)$ .

Similarly, we can obtain the calculability of the GIRI S for the BGI  $g_4, g_5, ..., g_M$  as part of the GI S.

If the GI S contains the generalized GIs  $G_1, G_2, G_3, ..., G_M$  or their combinations with the calculable RIs, joined by disjunction and conjunction signs, then their RI is calculable. Indeed, after calculating the GIRI  $G_1, G_2, G_3, ..., G_M$  and substituting them by the corresponding GIs  $g_0$ , the generalized GIs become BGI structures with simple elements for which the RI calculability is postulated.

The theorem is proved.

#### Conclusion

The purpose of writing this article is to justify one of the possible ways to obtain estimates of the indicators of the structural reliability of technical systems containing a large number of schemes with a large number of elements. The use of the universal graphoanalytical fault tree method, based on the full probability theorem, causes considerable difficulties from both the development of the algorithm and program, and the training of users.

The content of the article is directed at the development of algorithms and programs for calculating the reliability indicators of thermal power facilities, such as nuclear power plants, thermal power plants, directed systems for transporting water, oil, gas and other carriers, as well as for solving practical safety problems, planning repairs, ensuring the efficiency of system operation.

Currently, a large number of formulas, methods and calculation procedures for calculating the reliability indicators of widely used small-scale technical systems and their equipment have been developed, which led to the consideration of the method of typical structures in the article. The experimental computer verification of this approach has shown its effectiveness in several aspects: speed of data preparation, ease of use, a wide field for development and application.

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## Розрахунок показників надійності технічних систем методом типових структурних схем

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Наведено метод, розроблений для розрахунків показників структурної надійності систем з великим числом елементів. Метод ґрунтується на використанні типових структурних схем, що відбивають принципову схему зв'язків між елементами. Показано, як шляхом поповнення та об'єднання типових структур можна створювати графологічні структури для виконання розрахунків показників надійності. Підхід може бути використаний під час розробки алгоритмів і програм розв'язання на ЕОМ задач, що базуються на оцінках структурної надійності систем. До числа таких задач, зокрема, належать: оцінки безпеки атомних блоків, планування їх ремонтів, оцінки надійності спрямованих систем транспортування середовищ, оцінки залишкових ресурсів технічних об'єктів. Для їх розв'язання розроблені різні окремі методи. Однак стандартизувати розрахунки показників надійності неможливо через різноманітність систем і умов їхнього функціонування. Поданий підхід орієнтований на автоматизацію розрахунків показників структурної надійності широкого класу технічних систем. Він базується на доказі існування алгоритму розрахунку на множині типових структурних схем. Водночас передбачається, що на ЕОМ розпізнавані образи типових структур в складі графологічних образів систем. Зміст задачі полягає в такому. Є технічна система. Потрібно побудувати графологічний образ і розрахувати показник її структурної надійності. Метод розрахунку, що пропонується, трунтується на зображенні графологічного образу системи у вигляді композиції графологічних образів типових структур, показники надійності яких обчислювальні. Вони замінюються окремими елементами з обчисленими значеннями показника надійності. Заміни дають можливість спростити початковий графологічний образ системи за рахунок скорочення загального числа елементів і обчислити показник надійності системи. Процедура обчислення і заміни триває доти, поки в графологічному образі системи не залишиться одна типова структура, для якої показник надійності обчислювальний. Кількість елементів в системі нічим не обмежена, оскільки процедура замін здійснюється послідовно до створення однієї типової структури. Істотне обмеження в застосуванні методу до розрахунку структурної надійності широкого спектра складних технічних систем обумовлено обмеженістю множини типових структур. Однак такий банк типових структур може бути створений і використовуватися під час розробки відповідних розрахункових програм.

Ключові слова: алгоритм, система, структурна надійність, типові схеми.

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# MINIMIZATION OF THE STRESSED STATE OF A STRINGER PLATE WITH A HOLE AND RECTILINEAR CRACKS

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As is known, thin plates with holes are one of the most common structural elements. To increase their reliability and service life, it is of interest to find such a hole contour that ensures the minimum circumferential stress thereon, and also prevents the growth of possible cracks in the plate. This article deals with the problem of minimizing the stress state on the contour of a hole in an unbounded isotropic stringer plate weakened by two rectilinear cracks. Crack faces are considered to be free of stress. Determined is the optimal hole contour, at which no crack growth occurs, and the maximum circumferential stress thereon is minimal. The minimax criterion is used. The parameter characterizing the stress state in the vicinity of crack tips, according to the Irwin-Oroan theory of quasi-brittle fracture, is the stress intensity factor. The plate undergoes uniform stretching at infinity along the stringers. It is believed that the plate and the stringers are made of various elastic materials. The action of the stringers is replaced by the unknown equivalent concentrated forces applied at the points of their attachment to the plate. To determine these forces, Hooke's law is used. Applying the small parameter method, the theory of analytic functions and the method of direct solution to singular equations, we constructed a closed system of algebraic equations. This system depends on the mechanical and geometrical parameters of the plate and stringers, ensures the on-hole contour stress state minimization and equality of stress intensity factors to zero in the vicinity of crack tips. The minimization problem is reduced to a linear programming problem. The simplex method is applied.

*Keywords:* stringer plate, stress minimization, cracks, optimal hole contour, minimax criterion.

## Introduction

One of the most common structural elements is thin plates. Frequently, such plates have technological holes. Since the holes are stress concentrators and can lead to premature failure, the problem of minimizing the stress state on the hole contour is of great interest [1–15]. Article [1], based on the finite element method (FEM), develops an iterative method to optimize the hole contour to simultaneously minimize the tangential stresses in several areas around the hole boundary. It shows that such an optimal hole contour can significantly reduce peak stress in all the areas around the hole boundary, compared to typical non-optimal circular holes. Article [2] describes a piecewise-smooth optimal contour that minimizes local stresses under remote shear for a single, stress-free hole in an elastic plate, with the methods of conformal mapping and genetic algorithm used. It shows numerically that the hole contour found provides a shear stress by 30% lower than the stress concentration factor

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