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ADAPTIVE COMPUTATION OF CURVE LENGTHS GIVEN BY NON-DIFFERENTIABLE FUNCTIONS

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Measurement of the lengths of curves is quite common in solving various problems. If the function that defines the curve is differentiable, then computing the curve length is a relatively simple mathematical operation. In the absence of initial information about the function, it is necessary to apply approximate methods. Which of these methods should be used for a particular function is usually decided by the user. One of the important factors influencing the choice of the method is the available time resource for the preliminary analysis of the function and for the coordination with the initial data that include both the necessary accuracy of the result and the total numerical costs. The article proposes a method based on an a posteriori approach to the problem, where the analysis of the behavior of the function is carried out in the process of an approximate measurement of the length of the curve in a given area. This method became possible thanks to the introduction of an incremental adaptation mechanism that responds to the deviation of the function curve from the broken line approximating it. As a result, the local analysis accepted as a result of the adaptation made it possible to pass the large steepness segments of the curve in small increments and the flat segments, with large ones. With a particularly sharp change in the function (for example, in sub-domains with singularities), the main adaptation mechanism is able to go beyond the boundaries of the adopted set of constants without serious complications of the algorithm. Thus, there has disappeared the need both for a preliminary analysis of the behavior of the function, not necessarily regular, and the identification of singularities (kinks, extreme points, etc.), their numbers and locations. In order to compute the length of the curve, it is enough to set the function on this area and the required accuracy, limited by the minimum increment, without worrying about using some auxiliary tables and weight factors. The numerical experiment conducted on a test set of functions of varying complexity showed the advantage of the proposed approach over grid methods, especially with equally spaced nodes.

Keywords: *non-differentiable function, piecewise linear approximation, adaptive peace-wise selection of nodes, efficiency index.*

Introduction

Measurement of curve lengths is a very common operation that occurs, for example, in optimal control theory, isoperimetric analysis, geodetic constructions, and a number of related fields. This is a relatively simple operation, if the curve-defining function is differentiable. In the case of lack of information about the function or its differentiability, it becomes necessary to use approximate numerical methods.

When computing the length L of a line segment (Fig. 1) given by the function $f(x)$ on the segment [A, B], it is always important to choose the simplest and most economical search method.

In order to do this, we use the initial, most general definition, where the length of the line is measured as the limit of the sum of the lengths of the segments inscribed in it (line segments) with an unlimited increase in their number, when the maximum segment length tends to zero. Therefore, further, for an approximate computation of the line length, we will directly rely on the representation

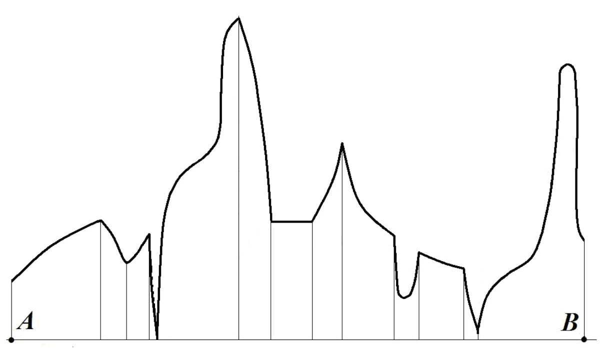

Fig. 1. A variant of the diverse behavior of a function

$$L = \sum_{k=1}^{n} l_k \,, \qquad (1)$$

where the lengths of the segments l_k of the broken line $g(x)$ inscribed in the curve are determined both by the nature of the function $f(x)$ and the method of specifying the nodes x_k that divide the segment $[A, B]$.

In order to solve problem (1), we consider it sufficient that the function be given by an analytical expression or some algorithm allowing us to uniquely indicate the number y for any number $x \in [A, B]$. The involvement of such functions, including non-differentiable ones, is justified by the fact that most real applied problems contain non-smooth functions [1–4], where no assumptions are made about the class of functions $f(x)$, except for the possibility of its computation at arbitrary points of a given segment $[A, B]$. In this case, the question arises both of the number n of segments in (1) and their rational arrangement on $[A, B]$ to ensure the required approximation accuracy ε.

Obviously, the more complete the initial information about $f(x)$, the more successful the solution to the problem assigned. Taking into account such a priori information as a function class, the presence and location of characteristic points, the behavior of the function in different regions of the segment $[A, B]$, as well as previous experience in solving such problems can be useful [5–12]. It has always been used to solve problems by classical methods.

According to the established tradition, with rare exceptions (Gauss K. F. [6, 7], Chebyshev P. L. [8]), for convenience, a grid of equally spaced nodes is used [5–7, 12], although it is clear that the way to choose them must depend on the behavior of the function $f(x)$ on $[A, B]$.

Adaptive Approach

The strive to reduce computational costs while ensuring a given accuracy, leads to an a posteriori way of solving such problems. In this case, local information is used on the change in the nature of the function $f(x)$ as it moves along the curve.

In order to obtain the length L (1) of the curve, we introduce a controlled incremental process

$$h_{k+1} = h_k U(Q_k(f, \varepsilon)) \,, \qquad (2)$$

generating nodes $\{x_k\}$ of segments l_k of the broken line $g(x)$ that approximates the true length of the curve L^* described by the function $f(x)$ on $[A, B]$. Here, U is a kind of adaptive control depending on the situation $\sigma_k = Q_k(f, \varepsilon)$ that occurs at the increment h_k with the corresponding segment l_k when the required accuracy ε is achieved by the given method of controlling the situation.

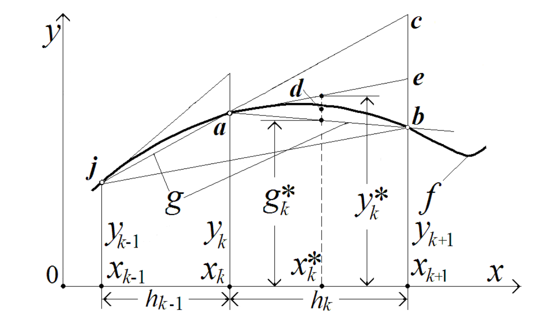

Fig. 2. A fragment of the scheme of deviation of g from f for adaptive increment control

Fig. 2 shows a fragment of the incremental construction of the broken line $g(x)$ ensuring the implementation of process (1), from which it follows that at least two ways are possible to track the changing situation σ_k along the curve described by the function $f(x)$.

In the first case, the situation

$$\sigma_k(a,b,c) = \overline{cb} = \left| y_{k+1} - (1-\mu_k) y_k + \mu_k y_{k-1} \right|, \quad \mu_k = h_k / h_{k-1} \,, \qquad (3)$$

is characterized by the deviation of the side $\overline{ac}$ from the side $\overline{ab}$ of the triangle Δ_{abc}. If the arc turns out to be a straight line, then the value $\sigma_k = 0$. With an increase in the curve knee, the value σ_k increases. It is clear that the smaller the angle $\angle a$, the more precisely the curve f approaches the broken line g.

Another possibility of controlling the "maximum" deviation of the chord $\overline{ab}$ from the arc $\overset{\frown}{ab}$ of the curve $f(x)$ (Fig. 2) is based on the most probable location of the deviation in the "middle" neighborhood of the arc $\overset{\frown}{ab}$, i.e. when the situation σ_k can be approximately characterized, for example, by the magnitude of the deviation

$$\sigma_k = 0{,}5\left|y_k^* - g_k^*\right| \tag{4}$$

Here, unlike the approximation method in [13], the approximate direction $\overline{ae}\,\|\,\overline{jb}$ of the "derivative" $f'(x)$ at the point (x_k, y_k) is taken into account, which allows us to more accurately approximate the point d (Fig. 2) to the arc $\overset{\frown}{ab}$ by using the informative points (x_{k-1}, y_{k-1}), (x_k, y_k), (x_{k+1}, y_{k+1}),

$$y_k^* = y_k + \lambda_k (y_{k+1} - y_{k-1}), \quad \lambda_k = 0{,}5\frac{h_k}{h_k + h_{k-1}}, \quad g_k^* = \frac{y_k + y_{k+1}}{2}, \quad x_k^* = \frac{x_k + x_{k+1}}{2}.$$

Hereinafter, informative points refer to the points where the values of the function are known.

Dependence (4) makes it possible, without computing the function $f(x)$ at the point x_k^*, to approximate the arc of the curve f of the broken line g composed of two segments $\overline{ad}$ and $\overline{db}$ whose total length is

$$l_k = 0{,}5\left(\sqrt{\left(y_k^* + g_k^* - 2y_k\right)^2 + h_k^2} + \sqrt{\left(y_k^* + g_k^* - 2y_{k+1}\right)^2 + h_k^2}\right). \tag{5}$$

In order to compute the approximate length L (1), we dwell on adaptive approach (2) for taking information by using σ_k (4), which makes it possible, by the formula

$$h_{k+1} = h_k \exp\left[\alpha(\varepsilon - \sigma_k)\right], \tag{6}$$

to obtain the lengths of increments, and from them, the lengths of segments l_k (5) through the quantities y_k^* and g_k^* that characterize the proximity of the broken line g to the curve f. The intensity factor α, which is part of (6), is responsible for the degree of increase or decrease in the increment change h_{k+1}, and ε, for the acceptable level of the deviation of y_k^* from g_k^*.

If the behavior of the function $f(x)$ on the current set x_k, x_{k+1} changes sharply, then not only the discriminant

$$D = \varepsilon - \sigma_k \tag{7}$$

turns out to be negative, but the rotation angle $\angle a$ of a segment of the broken line may become greater than a right angle ($\pi/2$). And then the next increment, determined by formula (6), may turn out to be too small, leading to a slowdown in the search process. Depending on the nature of the behavior of the singularity encountered, such an excessive deviation from the standard advancement along the curve will more or less negatively affect the amount of computational costs. Because of this, and also in order to eliminate the possible instability of the incremental process, we can introduce the minimum increment $h_{\min}$, i.e. agree that $h_k \geq h_{\min}$ for all numbers of k.

In the case of relatively flat curves, the incremental process, defined by formula (6), oscillates, and, where the deviations of the function $f(x)$ from $g(x)$ are large, it algorithmically switches to the "minimum increment" mode.

With a "sharp change in the direction of the function" according to the value of D (7), accuracy may be lost in calculating the length of the curve in the current segment (adaptive increment correction will occur only in the next segment). Therefore, in method (4)–(6), we can introduce the maximum tolerance $M>1$ for the deviation from ε, i.e. satisfy the condition $\sigma_k \leq M\,\varepsilon$. If it is violated, then it is necessary, within the framework of the incremental process defined by formulas (4)–(6), to introduce an additional informative point $x_k^* = x_k + h_k/2M$. This somewhat reduces the efficiency of the method, but it contributes to maintaining the deviation of g from f close to ε. After being adjusted and passing through a singularity, the process automatically switches to the previous mode. When the current increment h_{k+1} overlaps the boundary of the segment [A, B], then it is natural to take $x_{k+1}=B$ and complete the computation of L (1).

When adapting the increment [4, 13–22], we obtain a more accurate idea of the problem in the course of its solution, which is why the sensitivity to the initial data decreases. However, an unsuccessful choice of the initial increment h_0 can sometimes indirectly affect the error of the solution. Thus, a very small increment allows us to increase accuracy, but to a certain limit. However, at the same time, this increases numerical costs. An abrupt increment reduces the costs, but can lead to missing singularities at the very beginning. Given this uncertainty, it is necessary to limit the maximum and minimum increments, i.e. accept the condition $h_{\min} \le h_k \le h_{\max}$. But adaptation (6) partially solves the problem of the rational choice of the initial increment h_0, and makes it possible for the method to function even without specifying the beginning, relying only on $h_{\min}$ in the absence of other information.

Numerical Experiment

We can make correct conclusions about the capabilities of a particular method only after conducting appropriate tests on a representative set of a wide-class of test problems, and after comparing the results with those of the solutions obtained by other methods. It is desirable that the comparison take place on an equal footing with the methods that have been synthesized on the same basis (in this case piecewise-linear).

Certainly, in this process, an important role is played by the problems chosen for the experiment. If possible, they must reflect a fairly wide class of functions with a different spectrum of singularities (kinks, stationary points, etc.), and also rely either on well-known examples [5–7, 13, 10–19, 21, 22] or on the constructed ones, which combine diverse functions.

One such example of constructed functions may be the dependency

$$f(x) = \sum_{i=1}^{8} \chi(x - x_{i-1}) \cdot \chi(x_i - x) \cdot f_i, \qquad \chi(x) = (1 + \operatorname{sgn} x)/2,$$

composed of the set of functions $f_1 = 1$, $f_2 = \ln x + 1.693147181$,

$f_3 = \dfrac{0.955119613}{x - 1.0448803876}$, $f_4 = 1.791759469x - 2.583518938$,

$f_5 = 3.88089368 - 0.054221831 \cdot e^x$, $f_6 = -1$,

$f_7 = x\left(1.0(6) - 0.07(3)x\sqrt{6-x}\right)$, $f_8 = x^2 - 14.8(3)x + 53.(9)$,

that are defined on their segments whose ends are the junction points x_0=0, x_1=0.5, x_2=1.5, x_3=2, x_4=3, x_5=3.8, x_6=5, x_7=6, x_8=7.5 in which singularities (kinks, stationary points) are located. The behavior of the function $f(x)$ is illustrated in Fig. 3.

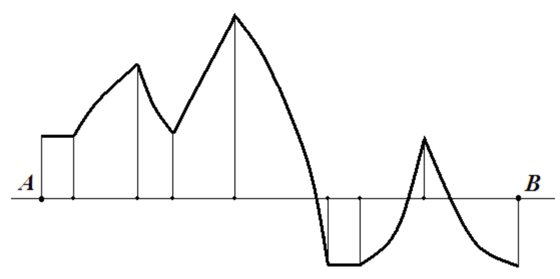

Fig. 3. An example of a combined function

The table below shows both the test functions and the results of calculating the lengths of their curves both by the adaptive exponential (*AdEx*) control method and, its special case, *Scan* method where there is no control, i.e. D=0. In this case, for the convenience of analyzing results, the number of informative points in the Scan method is chosen equal to the number obtained in the *AdEx* control method. The choice for the comparison of the *Scan* method rather than the more accurate Simpson or Gauss methods [6, 7, 12] is explained by the fact that the *Scan* method can give greater accuracy for functions with discontinuous derivatives (see Figs. 1, 3).

In assessing the quality of methods, the essential characteristics are usually both the laboriousness (solution time) of the problem and the amount of memory used. Therefore, in our case, we restrict ourselves to a conditional criterion (index of effectiveness) [23] in the form

$$E = (B - A)[N(|L - L^*| + \varepsilon)]^{-1}, \tag{8}$$

where the proximity measure $|L - L^*| \le \varepsilon$ on [A, B], L is the approximate solution (1), L^* is the exact solution, N is the number of computations of the function $f(x)$ to achieve the given accuracy ε . The length of the segment [A, B] in (8) serves as a leveling coefficient in many examples for the method being tested.

In the above table, the functions $f(x)$ and their corresponding approximations of the lengths L of curves are arranged in the increasing order of the complexity associated with the presence of singularities and their nature. The presence of singularities in the functions, as expected, somewhat reduces the efficiency of the E solution. But it still turns out to be higher than in the *Scan* method, which is based on the same piecewise-linear approximation of the function $f(x)$.

Computation Results of Test Cases

k	Function	$[A, B]$	Method	L	L^*	N	E
1	$(1-x^2)^{0.5}$	[-1; 1]	*Scan*	3.1412363916	3.141592653	177	8.331309
			AdEx	3.1418021985			9.341893
2	$(1-2.582843592\,x^2)^{0.5}$ [6]	**[0;0.6(2)]**	*Scan*	1.2912102895	1.291290325	85	6.777798
			AdEx	1.2912924346			7.304855
3	$(x^2)^{1/3}-x$	[-1; 2]	*Scan*	4.3674524180	4.408189781	255	0.281874
			AdEx	4.3891797505			0.587940
4	$e^{-x}\sin 2\pi x$	[0; 2]	*Scan*	4.1958229760	4.225902445	256	0.251372
			AdEx	4.2108289000			0.486047
5	$[1+(5x)^2]^{-1}$ [10]	[-1; 4]	*Scan*	6.0829017202	6.084423096	259	19.192283
			AdEx	6.0844301696			19.442536
6	$\sin 6x+\sin(7x+\pi)$	**[0;0.75π]**	*Scan*	11.225587396	11.227596321	215	3,642173
			AdEx	11.228236881			6,680063
7	$\mathrm{arctg}\,[(x-3)/(x^3+4)]$	[0; π/2]	*Scan*	5.0776795649	5.077680629	246	1,249688
			AdEx	5.0776823975			2.497811
8	$13(x-x^2)e^{-3x/2}$ [12]	[0; 4]	*Scan*	7.6622848307	7.662778876	239	33.814669
			AdEx	7.6630346397			65.694004
9	$\lvert x^4-5x^2+4\rvert$	[0; 2.5]	*Scan*	20.599563941	20.694115566	247	0.094415
			AdEx	20.697280762			2.187902
10	$\lvert\sin x\rvert+\lvert\ln x\rvert$	[0.1; 7]	*Scan*	9.4427715872	9.459615975	340	1.137283
			AdEx	9.4432600208			1.169288
11	$\left\lvert\pi^{-0.5}e^{(x-1)^2}-1{,}5\right\rvert+1{,}5$ [12]	[0; π]	*Scan*	56.859218061	56.886306843	340	0.348427
			AdEx	56.880234218			1.383769
12	$\lvert\sin x+\sin[(6e)^{1/4}x]\rvert$	[0; 6]	*Scan*	10.267133014	10.285142751	356	0.886594
			AdEx	10.272987199			1.281126
13	$[(x-0.3)^2+0.1^2]^{-1}+[(x-0.9)^2+0{,}2^2]^{-1}-6$ [3]	[0; 1]	*Scan*	192.79878747	192.805976090	518	0.235758
			AdEx	192.80520698			1.091234
14	$\lvert\sin(x^3/20)\rvert$	[0; 3π]	*Scan*	30.635488704	30.659352046	557	0.680544
			AdEx	30.656310079			4.186238
15	$\lvert\lvert x(x-4)\rvert-2\rvert$	[0; 6]	*Scan*	21.359827202	21.465187096	436	0.129386
			AdEx	21.431382222			0.395389
16	$f_1\chi(2-x)+\chi(x-2)[f_2\chi(5-x)+f_3\chi(x-5)]$, $f_1=\lvert(x-1)^4-1\rvert$, $f_2=\lvert(x-3)^4-1\rvert$, $f_3=(x-5)^4+15$	[-0.5;7]	*Scan*	42.180853268	42.191169149	557	1.856540
			AdEx	42.196942922			2.292393
17	$\sum_{i=1}^{8}\chi(x-x_{i-1})\cdot\chi(x_i-x)\cdot f_i$	[0; 7.5]	*Scan*	24.407974191	25.259851480	913	0.009632
			AdEx	25.361745241			0.079836
18	$10(15+2\sin x+10\cos x+3\cos 3x+$ $+3\sin 4x+\cos 5x)$ [7]	[-π; π]	*Scan*	689.81426576	689.822898510	1340	0.486768
			AdEx	689.82234612			3.020448

For all examples, the computation was carried out under the same conditions: $\varepsilon=10^{-3}$, $h_{\min}=10^{-3}$, $h_0=5\cdot10^{-3}$, $\alpha=10$, $M=2$.

The comparison of the curve length results obtained by the *Scan* and *AdEx* methods in the test cases shows that the efficiency criterion E for the *AdEx* method is higher in all test cases.

Conclusions

An adaptive method is proposed for numerically finding the length of a curve. This effective incremental method became possible due to the rejection of equally spaced nodes x_k, which are generally unfavorable for computing the integral sum at once over the entire segment $[A, B]$.

The adaptive local retrieval of the information regarding the nature of an arbitrary, not necessarily regular, function describing the shape of the curve made it possible to significantly reduce computational costs by keeping a uniform allowable deviation error ε over significantly extended segments $[A, B]$. There is no longer any need for any preliminary preparation or transformation of the function for the direct way of dividing the curve into segments with singularities. At the same time, computational costs have been reduced to almost optimal values. No a priori information is needed about the location and number of possible singularities, as well as their nature (kinks, stationary points, etc.). No special analysis is required to establish the value of the initial increment.

In order to implement the proposed method, it is enough to set the function $f(x)$ defined on $[A, B]$, the required accuracy ε, and the minimum allowable increment without any intermediate actions or using the tables of nodes and weight coefficients, and, furthermore, special functions associated with the established tradition of integration. The method is characterized by minimal time losses and amount of RAM used, which is especially important when solving problems that require strict time limits. At the same time, the method allows us to study the nature of the function and determine the areas with a sharp change in the values of the functions on $[A, B]$.

Naturally, the method equally shows its efficiency and effectiveness not only for special functions, but also in the case of ordinary continuous functions of a wide profile. The developed approach is universal in nature and seems promising for the use in various fields. The idea of the method easily extends to many cases of managing incremental procedures.

Using the *AdEx* method can be useful for many practical purposes (transport tasks: creating optimal networks (roads, pipelines, waterways, etc.), routing and tracing, cutting materials, and other technical and economic projects).

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Адаптивне обчислення довжин кривих, які задаються недиференційовними функціями

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Вимірювання довжин кривих є достатньо поширеним під час розв'язання різних задач. Якщо функція, що задає криву, є диференційовною, то обчислення довжини є досить простою математичною операцією. За відсутності початкової інформації про функцію доводиться застосовувати наближені методи. Який з цих методів за наявності конкретної функції доцільно використати, звичайно вирішує користувач, враховуючи клас функції та існуючий в його розпорядженні арсенал можливостей. Одним із важливих факторів, що впливають на вибір методу, є наявний ресурс часу на попередній аналіз функції та узгодження з початковими даними, які включають необхідну точність результату і загальні числові витрати. У статті пропонується метод, що ґрунтується на апостеріорному підході до проблеми, коли аналіз характеру поведінки функції здійснюється саме в процесі наближеного вимірювання довжини кривої в заданій області. Такий спосіб став можливим завдяки введенню покрокового адаптивного механізму, що реагує на відхилення кривої функції від її апроксимуючої ламаної. В кінцевому підсумку прийнятий локальний аналіз внаслідок адаптації дозволив проходити ділянки з великою крутістю кривої з малим кроком, а пологі – з великим. За особливо різкої зміни функції (наприклад, в підобластях з особливостями) основний адаптивний механізм наділений можливістю виходу за межі прийнятого набору констант без серйозних ускладнень алгоритму. Таким чином, відпала необхідність в попередньому дослідженні характеру поведінки функції, не обов'язково регулярній, та виявленні особливостей (зломи, екстремальні точки і т.п.), їх числа і місця. Для обчислення довжини кривої достатньо задати функцію на даній області і необхідну точність, обмежену мінімальним кроком, не піклуючись про використання якихось допоміжних таблиць та вагових коефіцієнтів. Проведений чисельний експеримент на тестовому наборі функцій різної складності показав перевагу запропонованого підходу над сітковими методами, особливо з рівновіддаленими вузлами.

Ключові слова: *недиференційовна функція, кусково-лінійне наближення, адаптивний покроковий вибір вузлів, індекс ефективності.*

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