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INTEGRAL CRITERION OF THE NON-UNIFORMITY OF STRESS DISTRIBUTION FOR THE TOPOLOGY OPTIMIZATION OF 2D-MODELS

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The emergence of new technologies for the production of structural elements gives impetus to the development of new technologies for their design, in particular with the involvement of a topology optimization method. The most common algorithm for designing topologically optimal structures is focused on reducing their elastic flexibility at a given volume of material. However, a closer to the engineering design approach is the minimization of the volume of a structural element while limiting the resulting mechanical stresses. In contrast to the classical algorithms of this approach, which limit the values of stresses at certain points, this paper develops an alternative criterion: the formation of the image of a structural element is based on minimizing the integral parameter of stress distribution non-uniformity. The developed algorithm is based on the method of proportional topology optimization, and when mechanical stresses are calculated, the classical relations of the finite element method are used. The above parameter can be interpreted as the ratio of the deviation of the values, ordered in ascending order, of equivalent von Mises stresses in the finite elements of a calculation model from their linear approximation to the corresponding mean value. The search for the optimal result is carried out for the full range of possible values of the averaged "density" of the calculation area, which is associated with a decrease in the amount of input data. The proposed integrated strength criterion provides better uniformity of the optimized topology, allows us to smooth the effect of the local peak values of mechanical stresses, determining a single optimization result that is resistant to calculation errors. The algorithm is implemented in the MatLab software environment for two-dimensional models. The efficiency of the approach is tested on the optimization of a classical beam (mbb-beam), a cantilever beam, and an L-shaped beam. A comparative analysis of the obtained results with those available in the literature is given. It is shown that in the absence of constraint on the average value of the density of a finite element model, the proposed criterion gives a "less dense" optimization result compared to the classical one (approximately 40%), while the values of "contrast index" are quite close.

Keywords: *topology optimization; two-dimensional problem; strength condition; integral criterion; algorithm; finite element method; equivalent von Mises stresses.*

Introduction

The term ″structural topology optimization″ was first introduced in [1], where an algorithm for the distribution of artificial composite material with the use of finite element method relations was presented. In this case, topology optimization is carried out in the absence of previous assumptions or information about the distribution of design quantities, and involves the optimization of both the shape of a structural element and inclusions in the form of cavities.

At the present stage, the emergence of new production technologies, including additives, as well as the requirements to reduce weight and cost give impetus to the development of new design technologies. This allows us to make structures lighter and stronger at the same time, which is reflected in design technologies, say, for aircraft structures, space and rocket hardware, and mechanical engineering [2].

When designing topologically optimal structures, there are two tasks, one of which is the task of reducing the elastic compliance of a structural element at a given volume of material, and the second one is aimed at minimizing the volume of material of this structural element while limiting the resulting mechanical stresses.

To solve the first task, the SIMP (Solid Isotropic Material with Penalization) method, which is called the penalty method, is used quite successfully. The effectiveness of the method has been confirmed by numerous publications, in particular [3–6]. The SIMP method is based on the partitioning of a design domain by finite elements (FE) with the introduction of a variable relative density ρ_i of their material ($0 \le \rho_i \le 1$: $\rho_i=0$ simulates the absence of an FE, $\rho_i= 1$ means a completely ″solid″ FE). In turn, the Young modulus of a FE is considered to be dependent on the power function of the relative density. A so called penalty is imposed on an FE with the current relative density $\rho_i' > 1$ or $\rho_i' \approx 0$, which allows us, in the process of optimization, to obtain a relative density close to one or zero (ie. leave the FE, or ″remove″ it).

The solution of topological optimization problems with account taken of stress constraints was presented in [7, 8], where the so-called evolutionary structural optimization (ESO) method is considered. According to this method, on the basis of finite element analysis (FEA), the stress distribution in a structure is determined, and a conclusion is made regarding the appropriateness of involving FEs with the use of the ″removal criterion″. This criterion uses the values of von Mises equivalent stresses both for the whole structure and for an individual FE. The FEs underloaded by a certain percentage are removed from the structure. The FEA cycle and the cycle of FE removal is repeated at one value of the ″removal criterion″ until a stable result is achieved. Then the value of the ″removal criterion″ increases, and the iteration procedure is repeated. The process is considered completed when all the FEs with the equivalent von Mises stress, which is a given percentage of the maximum, are removed. A disadvantage of this method is that the material removed in previous iterations, which could be used in the future, is not renewed. The Bi-Directional Evolutionary Structural Optimization (BESO) method does not have such a disadvantage [9]. In the BESO method, as a result of FEA, the displacements of the removed FEs are extrapolated, and their number (index of sensitivity) is determined as the change in the FE yield due to the removal or addition of FEs. After ranking all the FEs by sensitivity numbers, the FE structures "filled" with the material with the smallest sensitivity number are removed, and the "empty" ones with the maximum sensitivity number are returned to the structure.

As noted in [10], the analysis of mechanical stresses in topological optimization problems is closer to the engineering design approach. The problem of determining optimal topology is the problem of producing structures based on the application of the method of equal-strength structures.

Topology optimization, while limiting the value of the resulting mechanical stresses, is complicated by such problems as the nonlinearity of the corresponding problem and the phenomenon of singularity [3]. The latter is due to the fact that stress becomes uncertain in areas where the density is close to zero. This phenomenon of singularity is described in [11] in the topology optimization of truss structures with account taken of stress level constraints. This feature is caused by the emergence of sections where the cross-sectional area is close to zero.

One of the ways to apply stress constraints in the problem of structural topology optimization is the approach of local constraints on mechanical stresses at certain points [12]. Another approach is to transform these constraints into a single global constraint with the use of some aggregation function, such as the *p*-norm or the Kreisselmeier-Steinhauser function [13, 14].

Purpose of the Paper

This paper proposes an alternative criterion for forming the design of a structural element with the use of the integral parameter of the non-uniformity of stress state distribution, showing its effectiveness on specific examples. The developed algorithm is based on the method of proportional topological optimization with stress constraints, which is presented in [15].

Classical Formulation of the Topology Optimization Problem

The problem of topology optimization of a structural element with constraints on the level of its stress state is to minimize the conditional mass of the computational geometric domain of the element with account taken of strength condition fulfillment – the maximum mechanical stress values should not exceed a certain allowable value $[\sigma]$. This problem is usually solved in an automated mode, which involves the finite element method (FEM) to calculate the field of mechanical stress. And if the computational domain of the structural element is divided into N of FEs and mechanical stress, the "relative" density and volume (area in the case of 2D computational models) of the i-th FE of this domain is denoted by $\tilde{\sigma}_i$, ρ_i and V_i, accordingly,

the optimization problem can be written as

$$\begin{cases} \sum_i \rho_i V_i \to \min \ (i = \overline{1, N}); \\ \tilde{\sigma}_i \le [\sigma]. \end{cases} \tag{1}$$

The calculated variable in this problem is the ″relative″ density ρ of each element of the FE model ($0 \le \rho_{min} \le \rho_i \le 1$; i is the FE ordinal number; ρ_{min} is the minimum value of density), which is associated with the mass of this FE ($m_i=\rho_i V_i$) and also determines its ″relative″ elastic modulus E (Young's modulus). According to the modified SIMP method [3], the value of E depends on FE density

$$E(\rho) = E_{min} + \rho^p (E_0 - E_{min}),$$

where E_{min} is the minimum Young's modulus value, which characterizes ″empty″ FEs with $0 \le \rho \le \rho_{min}$; $E_0=1$; p is the ″penalty″ coefficient. Usually $E_{min}=10^{-9}$ and $p=3$.

The mechanical stress $\tilde{\sigma}_i$ in the i-th FE in expression (1) is usually the equivalent stress calculated in its geometric center in accordance with the energy theory of strength. In the case of a 2D calculation scheme, the specified stress is expressed in terms of the components of the mechanical stress tensor $\boldsymbol{\sigma} = \{\sigma_x \ \sigma_y \ \sigma_{xy}\}^T$ for the corresponding FE

$$\tilde{\sigma} = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\sigma_{xy}^2}\ . \tag{2}$$

The components **σ** depend on the vector **u** that contains the nodal displacements of FEs, and, according to the basics of solving FEM problems, is calculated by the formula

$$\boldsymbol{\sigma} = \mathbf{D}\,\mathbf{B}\mathbf{u}\,. \tag{3}$$

Here, **D** is the matrix of elastic constants (**D**=**D**(E, ν); ν is Poisson's ratio of the FE material); **B** is the matrix with derivatives of basis functions. For the plane stress state and bilinear functions of the form of a square FE of immeasurable unit length, both these matrices and the vector are defined as follows [15]:

$$\mathbf{D} = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix}; \quad \mathbf{B} = \frac{1}{2}\begin{bmatrix} -1 & 0 & 1 & 0 & 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 & 0 & 1 & 0 & 1 \\ -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 \end{bmatrix};$$

$$\mathbf{u} = \begin{Bmatrix} u_{2x} & u_{2y} & u_{4x} & u_{4y} & u_{3x} & u_{3y} & u_{1x} & u_{1y} \end{Bmatrix}^T.$$

In this case, the matrix **B** and vector **u** correspond to the coordinate system and numbering of FE nodes, which are shown in Fig. 1. It should be noted that this sequence of node numbers is quite common in the works that are devoted to the problems of topology optimization [4, 15].

The components of the vector **u**, in turn, are part of the global displacement vector **U** for the entire FE model, and are determined from a system of algebraic equations

$$\mathbf{KU}=\mathbf{F}, \tag{4}$$

where **K** is the global stiffness matrix, which, similarly to the vector **U**, is obtained as a result of combining elemental matrices; **F** is the vector of nodal efforts.

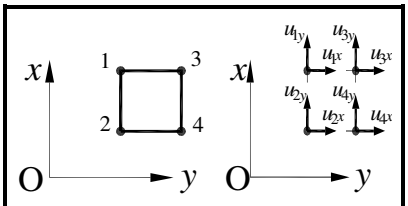

Fig. 1 Numbering of plane FE nodes

Based on the results FEA (3) and (4) of the stress-strain state of the structural element, the previous value of the ″relative″ density of each FE ρ_i' of the model is calculated as a linear combination of the density ρ_i^{prev} at the previous iteration and the ″optimal″ ρ_i^{opt} depending on the stress state of this FE. In particular, the following relations are used in the literature:

$$\rho_i' = \alpha \cdot \rho_i^{prev} + (1-\alpha) \cdot \rho_i^{opt}; \quad \rho_i^{opt} = R \frac{\tilde{\sigma}_i^q}{\sum_i \tilde{\sigma}_i^q}. \tag{5}$$

Here, α is a ″coefficient of history″ [15]: (α=0.5 means that the FE density ρ_i' in the current iteration will be the arithmetic mean of ρ_i^{prev} and ρ_i^{opt}); R is the residual amount of material (average density of the model); q is the exponent.

It should be noted that the coefficients and α and q are the main parameters [15] that determine the efficiency / convergence of the solution of topological optimization problem (1). However, the results of numerous calculations available in the literature indicate that the values α=0 and q=2 are appropriate. The first equation obviously does not take into account the element density from the previous iteration, and, accordingly, indicates no dependence on ″history″. The second one indicates the appropriateness of involving a quadratic proportion between ρ_i^{opt} and $\tilde{\sigma}_i$, ie the amount of material should be distributed among FEs proportional to the square of element stresses.

A mandatory and final step in each iteration, in determining the density distribution over a FE model, is the filtration of the previous value of the density ρ_i'. The said filtration allows avoiding jagged edges, narrow FEs, sharp boundaries, the so-called ″chessboard″ phenomena, etc. This filtration is, in essence, the local averaging of densities in the vicinity of a particular FE, with maintaining the value of the averaged density of the entire calculation area $\sum_i \rho_i V_i \Big/ \sum_i V_i$. The filtering algorithm itself is based on the following formula:

$$\rho_i = \sum_j w_{ij}\rho'_j \Big/ \sum_j w_{ij}\ ;\ 0 \le \rho_{min} < \rho_i \le 1. \tag{6}$$

Here, ρ_i is the filtered density of the i-th FE; w_{ij} is the weight function, which is inversely proportional to the distance between the i-th FE and adjacent FEs with numbers j

$$w_{ij} = \begin{cases} (r_0 - r_{ij})/r_0 & \text{при } r_{ij} < r_0; \\ 0 & \text{при } r_{ij} \ge r_0, \end{cases}$$

r_{ij} is the distance between the i-th and j-th FEs; r_0 is the filter radius.

Topology Optimization Algorithm

Most of topology optimization algorithms focused on checking the strength condition fulfillment, are based on the following procedures. At the first stage, the vectors **U**, **F** and matrices **D**, **B**, **K** are formed to implement FEA. Then the algorithm enters the main cycle, each iteration of which begins with the definition of the field of deformations (4) and mechanical stresses (3). Next, the equivalent stresses $\tilde{\sigma}_i$ (2) of FE model elements $(i = \overline{1, N})$ are calculated, and the stop criterion (1) is checked to make certain that the maximum value $\tilde{\sigma}_i$ is close to the allowable stress limit [σ] (max($\tilde{\sigma}_i$)=[σ]). If the condition is met, the iterative process ends. Otherwise, the algorithm continues to optimize the topology, and goes into the internal loop. The first step in this cycle is to refine the target amount of material (average density) R, which will be the new amount of model $\sum_i \rho_i V_i \Big/ \sum_i V_i$ material for the next iteration. If the maximum value of mechanical stress in the elements of the calculation system exceeds the allowable limit (max($\tilde{\sigma}_i$)>[σ]), ie the strength condition is not met, the current amount of material increases by a certain, pre-set factor Δ – $R = \sum_i \rho_i^{prev} V_i \Big/ \sum_i V_i + \Delta$ (here, $\sum_i \rho_i^{prev} V_i \Big/ \sum_i V_i$ is the current amount of material). Otherwise, the amount of material decreases with the same factor – $R = \sum_i \rho_i^{prev} V_i \Big/ \sum_i V_i - \Delta$. In the next step, the algorithm uses the iterative procedure (5) and (6) to distribute R among the main FEs.

The algorithm itself is shown in Fig. 2.

This sequence is implemented, in particular, in publication [15], where the authors proposed an algorithm for the proportional optimization whose effectiveness has been tested on a number of specific numerical examples.

- ♦ Adjustment of matrices and vectors for FEA and filtering
- ♦ Start of the main cycle of topology optimization
  - • Performance of FEA
  - • Check-up of stop criteria (interruption of the cycle, if they are fulfilled)
  - • Launch of the algorithm for determining the ″conditional″ density of FEs
    - • Determination of the target amount of material R
    - • If the strength condition is not met $R=\sum_i \rho_i^{prev} V_i \Big/ \sum_i V_i + \Delta$
    - • If the strength condition is met $R=\sum_i \rho_i^{prev} V_i \Big/ \sum_i V_i - \Delta$
    - • Reset of density data ($\rho_i=0$)
    - • Until $R-\sum_i \rho_i V_i \Big/ \sum_i V_i$ is small enough
      - – Identification of ρ_i^{opt} and distribution of ρ_i'
      - – Application of a filter to calculate the density ρ_i
      - – Update of the field of distribution of the density ρ_i

Fig. 2. Topology optimization algorithm

The main disadvantage of this approach is the necessity that the input data have an uncertain value of the allowable stress [σ], which is coordinated with the field of mechanical stress of the design structure during the action of a single force load. This disadvantage is due to the fact that for different calculation schemes, the values of [σ] can differ in a fairly wide range. Therefore, it is difficult to predict in advance the optimal [σ] with which the optimization problem will have a stable solution. Obviously, in this case, the determination and adjustment of [σ] must be done manually.

Another, more significant disadvantage of this class of algorithms, is the use of the maximum value of mechanical stress $\max(\tilde{\sigma}_i)$. Obviously, for most applied strength calculation problems, $\max(\tilde{\sigma}_i)$ is a reflection of the stress state in the vicinity of a particular stress concentrator, and, accordingly, does not determine the stress state of the design component as a whole, because it is quite local in nature. Thus, Fig. 3 shows the classical distribution of the mechanical stress $\tilde{\sigma}_i$ of the elements of a FE model with nonzero density ($\rho_i>\rho_{min}$), which are arranged in ascending order.

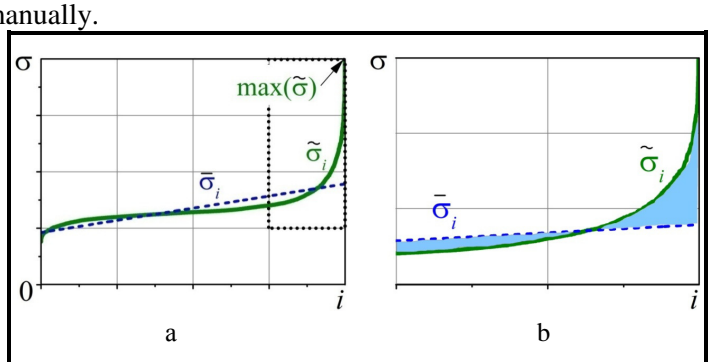

Fig. 3. Non-uniform distribution of mechanical stresses: a – for $\tilde{\sigma}_i>0$; b – for $\tilde{\sigma}_i \sim \max(\tilde{\sigma})$

It is obvious that $\max(\tilde{\sigma}_i)$ exceeds many times over the averaged value of the stress, which corresponds to the stresses in the central area of the figure (solid curve). When designing real components to avoid peak values of the field of mechanical stresses, and, accordingly, a significant reduction in their maximum values, a variety of design solutions and techniques are involved. Therefore, when implementing topological optimization, the focus on the criterion $\max(\tilde{\sigma}_i)=[\sigma]$ is not appropriate.

In this paper, we propose an alternative strength condition for system (1), which is focused on minimizing the non-uniformity of stress state distribution among FEs with the nonzero relative density ($\rho_i>\rho_{min}$). In particular, instead of the criterion $\max(\tilde{\sigma}_i)=[\sigma]$, the problem of finding the minimum of the function, $\sum_i |\tilde{\sigma}_i - \overline{\sigma}_i| \Big/ \sum_i |\overline{\sigma}_i| \to \min$, is set, and the optimization problem (1) will be written as

$$\begin{cases} \sum_i \rho_i V_i \Big/ \sum_i V_i \to \min; \\ \sum_i |\tilde{\sigma}_i - \overline{\sigma}_i| \Big/ \sum_i |\overline{\sigma}_i| \to \min. \end{cases} \quad (7)$$

Here, $\overline{\sigma}_i$ is a linear approximation of ordered values $\tilde{\sigma}_i$. The approximation coefficients are determined by the method of least squares.

It is obvious that the first condition of system (7) is related to the minimization of the average value of the density R (volume fraction) of the calculated region of the FE model of a component ($\mathrm{R} = \sum_i \rho_i V_i \Big/ \sum_i V_i$; $i = \overline{1, N}$). The second condition, from a geometric point of view, means minimizing the ratio of the area $\|\tilde{\sigma} - \overline{\sigma}\|_1$ of the region between the equivalent von Mises stresses $\tilde{\sigma}_i$ and their linear approximation $\overline{\sigma}_i$ to the area of the trapezoid $\|\overline{\sigma}\|_1$ under the line $\overline{\sigma}_i$. Here, $\|f\|_1$ is the norm of the vector f in the metric L^1 ($\|f\|_1 = \sum_i |f_i|$). The fragment of the area $\|\tilde{\sigma} - \overline{\sigma}\|_1$ is displayed in solid color in Fig. 3, b, which is essentially an enlarged image of the dotted zone in the right part of Fig. 3, a. The area of the trapezoid $\|\overline{\sigma}\|_1$ is equal to the product of the average value of the vector-column of stresses $\overline{\sigma}$ and its height.

If we enter the parameter $X = \|\tilde{\sigma}_i - \overline{\sigma}_i\|_1 / \|\overline{\sigma}_i\|_1$, problem (7) can be written in a more compact form:

$$\begin{cases} \mathrm{R} \to \min; \\ X \to \min. \end{cases} \quad (8)$$

Numerical Results

Specific calculations were performed for 2D calculation schemes, which were considered in [15]. In particular, for the models of a classical beam (so-called mbb-beam), a cantilever beam, and an *L*-beam (see column 1 in Fig. 4), which are under single force loads (P=1). The dimensions of these models are determined by a dimensionless parameter s=40 (FEs are squares with a unit length of sides, V_i=1).

The results of topological optimization based on criterion (1), which are depicted in column 2 (Fig. 4), were obtained by the authors of [15]. It should be noted that the limit for the maximum value of stress [σ] for the first beam in this work is taken equal to 1.08; for the second beam, 0.57, and for the third beam, 1.05. Also during FEA, the external load was distributed on several adjacent FEs to improve the convergence of the computational process.

The results of the topology optimization of the data of calculation schemes using criterion (8) are shown in column 3 (Fig. 4).

From the comparison of the optimization results, it can be concluded that in the absence of constraint on the average value of the density R of a FE model, criterion (8) in comparison with conditions (1) gives a ″less dense″ result (approximately 40%). Quantitative indicators for both criteria are summarized in the table, which also shows the values of the so-called ″contrast index″ [15], which characterizes the proportion of FEs with the boundary values of relative density ($\rho < \rho_{min}$ i $\rho \geq_{max}$)

$$C = 1 - \frac{N_\rho}{N},$$

where N_ρ is the number of FEs with the density $\rho_{min} < \rho < 1 - \rho_{min}$ (ρ_{min}=0.01).

It is obvious that in contrast to R the contrast indices C for both optimization criteria differ insignificantly.

The average density of optimized structural elements and their contrast index C

Scheme	Density R		Index *C*	
	Criteria (1)	Criteria (8)	Criteria (1)	Criteria (8)
mbb-beam	0.31	0.20	0.83	0.812
cantilever beam	0.34	0.16	0.88	0.856
L-beam	0.33	0.21	0.85	0.848

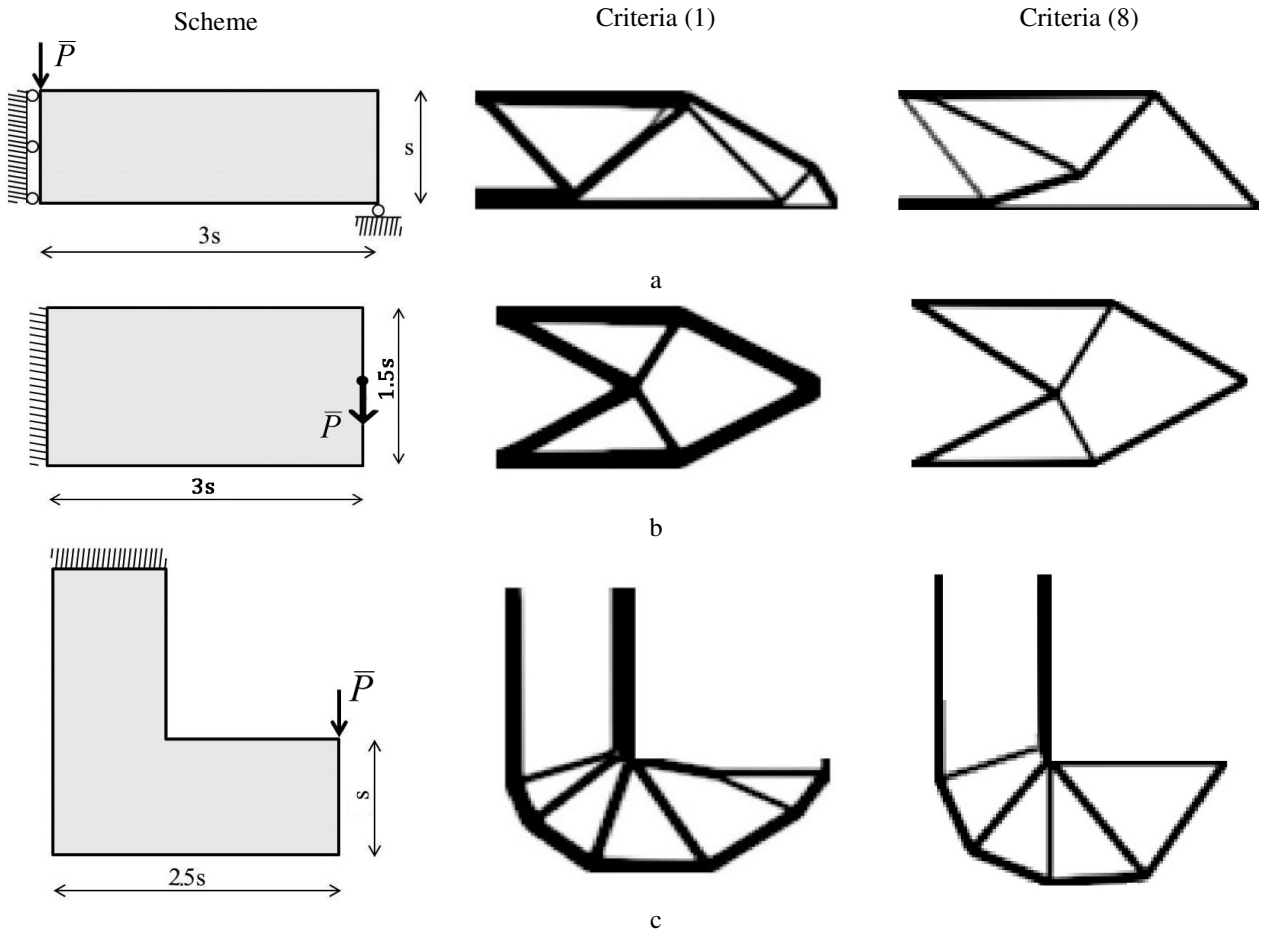

Fig. 4. Topological optimization of 2D-models:
a – mbb-beam; b – cantilever beam; c – L-beam

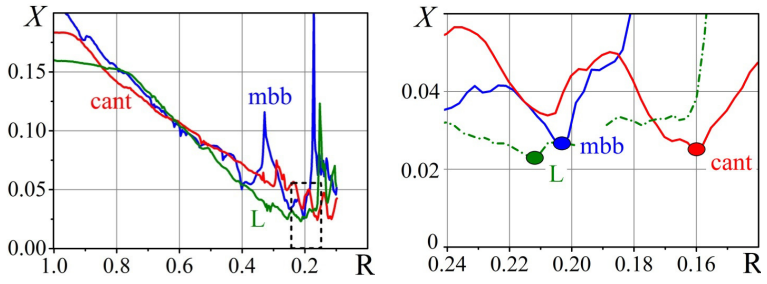

Fig. 5. Graph of the non-uniformity of the distribution of the stress state:
a – for R=1...0.1; b – for R=0.24...0.14

It should be noted that the optimization results in column 3 (Fig. 4) correspond, in essence, to the minimum of the function $X=X(R)$, because the numerical calculations showed that in the whole range of possible values of R (volfrac≤R≤1) the function $X=X(R)$ has a single minimum. The graphs of these functions for the three calculation schemes considered above are shown with the curves of the same name in Fig. 5. Fig. 5, b is an enlarged rectangular fragment, which is limited in Fig. 5, a with a dashed border.

In Fig. 5, b, the circles show the minima of the graphs corresponding to the solutions of topological optimization problem (8), shown in column 3 (Fig. 4). It should be noted that the graphs in Fig. 5, a are constructed in the mode of linear reduction of R from 1 to volfrac [15], however, the choice of the initial value R_0 is not fundamental (volfrac≤R_0≤0.5 is usually taken to speed up calculations).

A disadvantage of the criterion (8) presented in this paper is the need to solve the problem of finding the optimal geometry over the entire range of possible values of the averaged density R, which slightly increases the time of solving topological optimization problem (8) compared to the classical formulation (1). However, scanning occurs with a minimum amount of input data, and in the absence of the need for predetermination [σ], which indicates the advantages of this criterion. The criterion $X(R)\rightarrow\min$ is also an integral criterion, so it allows us to obtain a solution that is more resistant to errors in the input data and inaccuracies in calculations.

Conclusions

The paper presents a new criterion for the topological optimization of structural elements, which is focused on minimizing the non-uniform distribution of their stress state. In contrast to existing topological optimization algorithms based on the classical strength condition, the proposed algorithm uses an integrated stress assessment, which reduces the influence of local peak stress values, and provides better uniformity of the design component. This integral criterion can be interpreted as the ratio of the deviation of the ordered values of equivalent von Mises stresses from their linear approximation to the mean value. The disadvantages of this method include the need to find the optimal result over the entire range of possible values of the average density of the calculation domain. However, the main advantages of this method include a reduced amount of input data, ensuring greater uniformity of the optimized topology, the presence of a single optimization result, insensitivity to calculation errors. The effectiveness of the criterion is confirmed by numerical examples with traditional calculation schemes. The developed approach can be developed to solve the problems of topological optimization in a three-dimensional formulation, and generalized in the case of optimization of a structural element by the criterion of its fatigue strength.

References

1. Bendsøe, M. P. & Kikuchi, N. (1988). Generating optimal topologies in structural design using a homogenization method. *Computer Methods in Applied Mechanics and Engineering*, vol. 71, iss. 2, pp. 197–224. https://doi.org/10.1016/0045-7825(88)90086-2.
2. Borovikov, A. A. & Tenenbaum, S. M. (2016). *Topologicheskaya optimizatsiya perekhodnogo otseka kosmicheskogo apparata* [Topology optimization of spacecraft transfer compartment]. *Aerokosmicheskiy nauchnyy zhurnal – Aerospace Scientific Journal*, vol. 2, no. 5, pp. 16–30. (in Russian). https://doi.org/10.7463/aersp.0516.0847780.
3. Bendsoe, M. P. & Sigmund, O. (2004). Topology optimization: theory, methods and applications. Berlin: Springer-Verlag, 370 p. https://doi.org/10.1007/978-3-662-05086-6_2.
4. Sigmund, O. (2001). A 99 line topology optimization code written in Matlab. *Structural and Multidisciplinary Optimization*, vol. 21, iss. 2, pp. 120–127. https://doi.org/10.1007/s001580050176.
5. Andreassen, E., Clausen, A., Schevenels, M., Lazarov, B. S., & Sigmund, O. (2011). Efficient topology optimization in MATLAB using 88 lines of code. *Structural and Multidisciplinary Optimization*, vol. 43, iss. 1, pp. 1–16. https://doi.org/10.1007/s00158-010-0594-7.
6. Liu, K. & Tovar, A. (2014). An efficient 3D topology optimization code written in MatLab. *Structural and Multidisciplinary Optimization*, vol. 50, iss. 6, pp. 1175–1196. https://doi.org/10.1007/s00158-014-1107-x.
7. Xie, Y. M. & Steven, G. P. (1993). A simple evolutionary procedure for structural optimization. *Computers & Structures*, vol. 49, iss. 5, pp. 885–896. https://doi.org/10.1016/0045-7949(93)90035-C.
8. Huang, X. & Xie, Y. M. (2010). Evolutionary topology optimization of continuum structures: methods and applications. UK: Wiley, 223 p. https://doi.org/10.1002/9780470689486.
9. Xia, L., Xia, Q., Huang, X., & Xie, Y. M. (2018). Bi-directional evolutionary structural optimization on advanced structures and materials: A comprehensive review. *Archives of Computational Methods in Engineering*, vol. 25, iss. 2, pp. 437–478. https://doi.org/10.1007/s11831-016-9203-2.
10. Cysoyeva, V. V. & Chedrik, V. V. (2011). *Algoritmy optimizatsii topologii silovykh konstruktsiy* [Optimization algorithms for the topology of power structures]. *Uchenyye zapiski TSAGI – Central Aerohydrodynamic Institute scientific notes*, Vol. XLII, no. 2, pp. 91–101 (in Russian).
11. Kirsch, U. (1990). On singular topologies in optimum structural design. *Structural and Multidisciplinary Optimization*, vol. 2, iss. 3, pp. 133–142. https://doi.org/10.1007/BF01836562.
12. Duysinx, P. & Bendsøe, M. P. (1998). Topology optimization of continuum structures with local stress constraints. *International Journal for Numerical Methods in Engineering*, vol. 43, pp. 1453–1478. https://doi.org/10.1002/(SICI)1097-0207(19981230)43:8<1453::AID-NME480>3.0.CO;2-2.

13. Le, C., Norato, J., Bruns, T., Ha, C., & Tortorelli, D. (2010). Stress based topology optimization for continua. *Structural and Multidisciplinary Optimization*, vol. 41, iss. 4, pp. 605–620. https://doi.org/10.1007/s00158-009-0440-y.
14. Lee, E., James, K. A., & Martins, J. R. (2012). Stress-constrained topology optimization with design-dependent loading. *Structural and Multidisciplinary Optimization*, vol. 46, iss. 5, pp. 647–661. https://doi.org/10.1007/s00158-012-0780-x.
15. Biyikli, E. & To, A. C. (2015). Proportional Topology Optimization: A new non-gradient method for solving stress constrained and minimum compliance problems and its implementation in MATLAB. *PLoS ONE*, no. 10, pp. 1–18. https://doi.org/10.1371/journal.pone.0145041.

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Інтегральний критерій нерівномірності розподілу напруженого стану при топологічній оптимізації 2D-моделей

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Поява нових технологій виробництва конструктивних елементів дає поштовх до розвитку нових технологій їх конструювання, зокрема, із залученням методу топологічної оптимізації. Найбільш розповсюджений алгоритм проєктування топологічно оптимальних конструкцій орієнтований на зменшення їх пружної податливості при заданому об'ємі матеріалу. Разом з тим більш близькою до інженерного підходу у проектуванні є мінімізація об'єму конструктивного елемента при одночасному обмеженні виникаючих механічних напружень. На відміну від класичних алгоритмів такого підходу, що обмежують значення напружень в певних точках, в даній роботі розвинуто альтернативний критерій – формування образу конструктивного елемента здійснюється на основі мінімізації інтегрального параметра нерівномірності розподілу напруженого стану. В основу розробленого алгоритму покладено метод пропорційної топологічної оптимізації, а при обчисленні механічних напружень застосовані класичні співвідношення методу скінченних елементів. Зазначений вище параметр може бути інтерпретований як відношення відхилення впорядкованих у порядку зростання значень еквівалентних за Мізесом напружень у скінченних елементах розрахункової моделі від лінійної їх апроксимації до відповідного середнього значення. При цьому пошук оптимального результату здійснюється для усього діапазону можливих значень осередненої «густини» розрахункової області, що пов'язано зі зменшенням кількості вхідних даних. Запропонований інтегральний критерій міцності забезпечує кращу рівноміцність оптимізованої топології, дозволяє згладжувати вплив локальних пікових значень механічних напружень і визначає єдиний результат оптимізації, який є стійким до похибок при обчисленнях. Алгоритм реалізовано у програмному середовищі MatLab для двовимірних моделей. Ефективність підходу апробована на оптимізації класичної балки (mbb-балки), консольної балки і L-балки. Наведено порівняльний аналіз отриманих результатів з наявними у літературі. Показано, що за відсутності обмеження на осереднене значення густини скінченно-елементної моделі запропонований критерій дає «більш легкий» результат оптимізації у порівнянні з класичним (приблизно на 40%), водночас значення «індексу контрастності» є досить близькими.

Ключові слова: *топологічна оптимізація; двовимірна задача; умова міцності; інтегральний критерій; алгоритм; метод скінченних елементів; еквівалентні за Мізесом напруження.*

Література

1. Bendsøe M. P., Kikuchi N. Generating optimal topologies in structural design using a homogenization method. *Computer Methods Appl. Mech. and Eng*. 1988. Vol. 71. Iss. 2. P. 197–224. https://doi.org/10.1016/0045-7825(88)90086-2.
2. Боровиков А. А., Тененбаум С. М. Топологическая оптимизация переходного отсека КА. *Аэрокосм. науч. журн.* 2016. № 2. С. 16–30. https://doi.org/10.7463/aersp.0516.0847780.
3. Bendsoe M. P., Sigmund O. Topology optimization: theory, methods and applications. Berlin: Springer-Verlag, 2004. 370 p. https://doi.org/10.1007/978-3-662-05086-6_2.
4. Sigmund O. A 99 line topology optimization code written in Matlab. *Structural and Multidisciplinary Optimization*. 2001. Vol. 21. Iss. 2. P. 120–127. https://doi.org/10.1007/s001580050176.

5. Andreassen E., Clausen A., Schevenels M., Lazarov B. S., Sigmund O. Efficient topology optimization in MATLAB using 88 lines of code. *Structural and Multidisciplinary Optimization*. 2011. Vol. 43. Iss. 1. P. 1–16. https://doi.org/10.1007/s00158-010-0594-7.
6. Liu K., Tovar A. An efficient 3D topology optimization code written in MatLab. *Structural and Multidisciplinary Optimization*. 2014. Vol. 50. Iss. 6. P. 1175–1196. https://doi.org/10.1007/s00158-014-1107-x.
7. Xie Y. M., Steven G. P. A simple evolutionary procedure for structural optimization. *Computers & Structures*. 1993. Vol. 49. Iss. 5. P. 885–896. https://doi.org/10.1016/0045-7949(93)90035-C.
8. Huang X., Xie Y. M. Evolutionary topology optimization of continuum structures: methods and applications. UK: Wiley, 2010. 223 p. https://doi.org/10.1002/9780470689486.
9. Xia L., Xia Q., Huang X., Xie Y. M. Bi-directional evolutionary structural optimization on advanced structures and materials: A comprehensive review. *Archives Computational Methods in Eng*. 2018. Vol. 25. Iss. 2. P. 437–478. https://doi.org/10.1007/s11831-016-9203-2.
10. Сысоева В. В., Чедрик В. В. Алгоритмы оптимизации топологии силовых конструкций. *Уч. зап. Центр. аэрогидродинам. ин-та*. 2011. Т. XLII. № 2. С. 91–101.
11. Kirsch U. On singular topologies in optimum structural design. *Structural and Multidisciplinary Optimization*. 1990. Vol. 2. Iss. 3. P. 133–142. https://doi.org/10.1007/BF01836562.
12. Duysinx P., Bendsøe M. P. Topology optimization of continuum structures with local stress constraints. *Int. J. for Numerical Methods in Eng*. 1998. Vol. 43. P. 1453–1478. https://doi.org/10.1002/(SICI)1097-0207(19981230)43:8<1453::AID-NME480>3.0.CO;2-2.
13. Le C., Norato J., Bruns T., Ha C., Tortorelli D. Stress based topology optimization for continua. *Structural and Multidisciplinary Optimization*. 2010. Vol. 41. Iss. 4. P. 605–620. https://doi.org/10.1007/s00158-009-0440-y.
14. Lee E., James K. A., Martins J. R. Stress-constrained topology optimization with design-dependent loading. *Structural and Multidisciplinary Optimization*. 2012. Vol. 46. Iss. 5. P. 647–661. https://doi.org/10.1007/s00158-012-0780-x.
15. Biyikli E., To A. C. Proportional Topology Optimization: A new non-gradient method for solving stress constrained and minimum compliance problems and its implementation in MATLAB. *PLoS ONE*. 2015. No. 10. P. 1–18. https://doi.org/10.1371/journal.pone.0145041.