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OPTIMUM DESIGN OF REINFORCED CYLINDRICAL SHELLS UNDER COMBINED AXIAL COMPRESSION AND INTERNAL PRESSURE

Heorhii V. Filatov
gvmfilatov@gmail.com
ORCID: 0000-0003-4526-1557

State Higher Educational Institution "Ukrainian State University of Chemical Technology",
8, Haharina St., Dnipro, 49005, Ukraine

This paper discusses the use of the random search method for the optimal design of single-layered rib-reinforced cylindrical shells under combined axial compression and internal pressure with account taken of the elastic-plastic material behavior. The optimality criterion is the minimum shell volume. The search area for the optimal solution in the space of the parameters being optimized is limited by the strength and stability conditions of the shell. When assessing stability, the discrete rib arrangement is taken into account. In addition to the strength and stability conditions of the shell, the feasible space is subjected to the imposition of constraints on the geometric dimensions of the structural elements being optimized. The difficulty in formulating a mathematical programming problem is that the critical stresses arising in optimally-compressed rib-reinforced cylindrical shells are a function of not only the skin and reinforcement parameters, but also the number of half-waves in the circumferential and meridional directions that are formed due to buckling. In turn, the number of these half-waves depends on the variable shell parameters. Consequently, the search area becomes non-stationary, and when formulating a mathematical programming problem, it is necessary to provide for the need to minimize the critical stress function with respect to the integer wave formation parameters at each search procedure step. In this regard, a method is proposed for solving the problem of optimally designing rib-reinforced shells, using a random search algorithm whose learning is carried out not only depending on the objective function increment, but also on the increment of critical stresses at each extremum search step. The aim of this paper is to demonstrate a technique for optimizing this kind of shells, in which a special search-system learning algorithm is used, which consists in the fact that two problems of mathematical programming are simultaneously solved: that of minimizing the weight objective function and that of minimizing the critical stresses of shell buckling. The proposed technique is illustrated with a numerical example.

Keywords: *reinforced cylindrical shell, optimal design, random search, critical buckling stresses.*

Introduction

In the optimal design of building structures, which are complex multi-parameter systems with non-linear objective functions and non-linear constraints, the use of regular methods such as the Gauss-Seidel method, gradient and steepest descent methods, despite their high accuracy, is usually associated with severe computational difficulties. Sometimes they do not give a solution at all, while the complexity of the objective function and constraints does not cause significant difficulties in using stochastic programming methods.

With the development of computer technology, it became possible to create stochastic models of the objects of optimization, as well as analyze them, search and provide a sorting-through with account taken of the previous history of searching for models that satisfy a given optimality criterion. Modeling various optimization objects, in particular building structures, based on the use of random and pseudo-random numerous sequences was the basis of the optimization method referred to as random search [1].

Fundamental research in the field of random search methods is closely related to the works of L. A. Rastrigin [1, 2], L. S. Gurin, Ya. Z. Dymarsky, A. D. Merkulov [3], I. N. Bocharov, A. A. Feldbaum [4], V. Ya. Katkovnik [5], A. Fiakko, McCormic [6], D. Himmelblau [7], S. H. Brooks [8], D. C. Karnopp [9], M. Shumer & K. Stejglitz [10].

L. A. Rastrigin's works [1, 2] are devoted to the theory of statistical search methods. These works investigate the local properties of various random search algorithms, mainly those that adapt locally, provide estimates of the effectiveness of these algorithms, including random search algorithms with self-learning and forgetting, algorithms with a director cone, a director sphere, and a number of others.

As follow-up to L. A. Rastrigin's research in [11], algorithms using smoothing operators in problems of extremum search are considered; step adaptation in random search algorithms, and an algorithm for estimating descent directions, using the statistical gradient method, are considered; some global optimization algorithms and ravine random search algorithms are proposed, etc.

The use of the random search method in the optimal structural design is given in [12–21]. In practice, the random search method was used in the optimal design of the dynamic systems of the superstructure of the ERP-2500 bucket-wheel excavator in collaboration with the UkrNIIProekt of the Ministry of Coal Industry of the USSR [22]. The effect was found to be equal to 5% of the superstructure weight, which amounted to five tons.

Since the second half of the 1980s, random search methods have been used in the problems of optimal design of structures interacting with an aggressive environment. The results of optimization of such structures are described in detail in [23]. Random search, as a method for finding extreme solutions, was also used to develop an evolutionary theory of identification of mathematical models of corrosion destruction [24].

During the past five years, the random search method has been used in [25–28].

The experience of research, design, and operation shows that thin-walled shells reinforced by a system of ribs are the most rational ones in terms of weight. The load-bearing capacity of such shells is much higher than that of smooth unreinforced shells with the same wall thickness. However, the computation of shells reinforced by a system of ribs is much more complicated. The critical stresses arising in optimal compressed reinforced cylindrical shells are a function of not only the skin and reinforcement parameters, but also the number of half-waves in the circumferential and meridional directions that are formed due to buckling. In turn, the number of these half-waves depends on the variable shell parameters. Consequently, the search area becomes non-stationary, and when formulating a mathematical programming problem, it is necessary to provide for the need to minimize the critical stress function with respect to the integer wave formation parameters at each search procedure step.

Let us formulate the general statement of the problem of optimally designing a compressed reinforced shell, taking into account the above, in the form of the following mathematical model [29]:

$$\min_{\{K_j\}}\left[\min_{\{A_i\}} F(A_i)\right] \tag{1}$$

in satisfying the constraints

$$B_i(A_i)\ge 0,\quad \min_{(S)}\min_{(m,n)}\left[\tilde{B}_i(A_i)\right]\ge 0 \tag{2}$$

$$A_i^L \le A_i \le A_i^V,\quad (i=1,2,...,l_1-1,l_2+1,...,l;\ \ j=l_1,l_2;\ \ S=1,2,....,S_1), \tag{3}$$

where F is the objective function; A_i is the vector of optimization parameters; A_i^U, A_i^V are, respectively, the lower and upper boundaries of the change in the geometric parameters of the shell being optimized; m, n are the integer parameters of wave formation in the circumferential and meridional directions; K_j is the vector of discrete quantities that quantitatively characterizes the primary structure that reinforces the shell; S_i is the value corresponding to a certain type of structural deformation.

In general, model (1)–(3) has some specific features:

– the functions $B_i^{(S)}(A_i)=\min\limits_{(m,n)}\tilde{B}_j(A_i)$ "breathe" according to the integer parameters of wave formation m and n;

– the quantities m and n are associated with a certain dependence on the quantities K_j that discretely change, as a result of which the function $B_i^{(S)}(A_i)$ has a ravine character;

– the minimization using S, corresponding to the non-stationarity of constraints, leads to the fact that the zones of permissible changes in variable parameters can intersect, and, as a consequence, the appearance of local minima, as well as fractures or deformations of existing ravines is possible.

As follows from the above, at each search procedure step, a multi-extremal problem of mathematical programming is solved. In order to solve this problem, it is proposed to apply random search algorithms from the class of independent ones, for example, an independent global search algorithm with adaptation of the sample distribution or from the class of wandering algorithms, such as, for example, an algorithm with a director cone or sphere [1].

Statement of the Problem of Optimal Design of a Stringer-reinforced Cylindrical Shell

Consider a hinged edge-supported stringer-reinforced cylindrical shell of a given length L and radius R made of an ideal elastic-plastic material and loaded by the combined action of the axial compressive force N and internal pressure q (Fig. 1).

The axial force acts in the form of the uniformly distributed stresses p at the shell edges.

The assumption of the ideal elastic-plastic work of the shell material allows one to estimate the limiting state of structures in terms of strength in accordance with the maximum shear stress theory, and consider shell stability in linear formulation with the elastic behavior of the material. The rib eccentricity relative to the median skin surface is not taken into account.

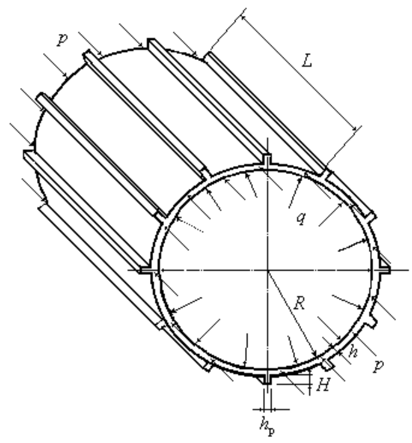

Fig. 1. Design scheme of a stringer-reinforced shell

Note that a similar design scheme can also be used in the optimal design of a ribbed-lattice shell (Fig. 2).

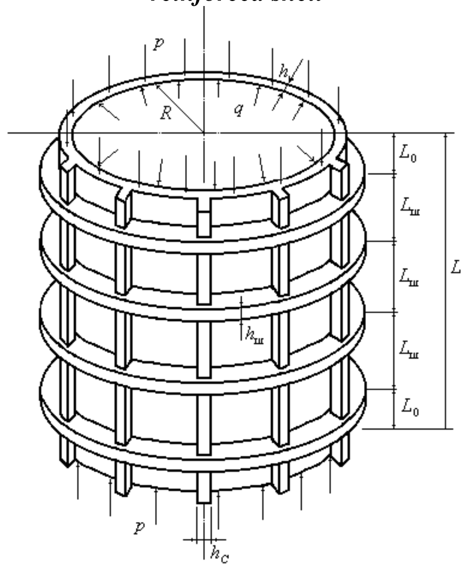

Fig. 2. Design model of a ribbed-lattice shell

In this case, the compartment between two frames is considered to be a hingedly supported stringer-reinforced shell under the assumption that the rigidity of the frames during torsion is small, and during bending, it is sufficiently large.

The meridional and circumferential stresses in the skin and the longitudinal stresses in the stringers under the combined action of the axial force and internal pressure are written as follows [29]

$$\sigma_1 = \frac{\overline{N} - \mu R q \gamma}{h(1+\gamma)}; \quad \sigma_2 = \frac{\overline{N} + \mu R q}{h(1+\gamma)}; \quad \sigma_3 = -\frac{qR}{h}, \tag{4}$$

where $\overline{N} = \dfrac{N}{2\pi R}$; γ is the ratio of the cross-sectional areas of the stringers and skin; μ is Poisson's ratio, q is the value of the internal shell pressure; R is the radius of the middle shell surface; h is the shell-wall thickness.

We will assume that the limiting state of the shell will be determined using the condition of skin yield according to the theory of the highest shear stresses

$$\sigma_{\text{equiv}} = \sigma_1 - \sigma_3 \le [p], \tag{5}$$

where $[p]$ is the yield point.

Substituting the equations for σ_1 and σ_3 into expression (5), we obtain the strength condition for the stringer-reinforced shell

$$2\pi R\{h[p](1+\gamma) - Rq(1+\gamma) + \mu R q \gamma\} \ge N. \tag{6}$$

The required value of the critical load, which avoids the buckling of the stringer-reinforced shell at a given load, is provided by longitudinal reinforcement.

To assess stability, formulas are used for the parameters of the critical stresses of structurally anisotropic shells with account taken of the discrete arrangement of the ribs [30]:

1) a general case of buckling, when the stringers bend and twist

$$\eta_1 = \frac{1}{1+\gamma k}\left[U + \frac{t}{12\langle 1-\mu^2\rangle}\left(\alpha k \tilde{m}^2 + \beta k n^2\right) + \tilde{q}\,\frac{n^2}{\tilde{m}^2}\right] \tag{7}$$

2) a special case of buckling, when the stringers only bend

$$\eta_2 = \frac{1}{1+2\gamma k}\left\{U + \frac{t\alpha k\tilde{m}^2}{6(1-\mu^2)} + \tilde{q}\frac{n^2}{\tilde{m}^2}\right\} \tag{8}$$

3) a special case of buckling, when the stringers only twist

$$\eta_3 = U + \frac{t\beta k n^2}{6(1-\mu^2)} + \tilde{q}\frac{n^2}{\tilde{m}^2}, \tag{9}$$

where $t=\frac{h}{R}$; $\gamma=\frac{F}{2\pi Rh}$; $\alpha=\frac{EJ}{2\pi RD}$; $\beta=\frac{GJ_{cr}}{2\pi RD}$; $D=\frac{Eh^3}{12(1-\mu^2)}$; $\tilde{m}=\frac{m\pi R}{L}$; $G=\frac{E}{2(1+\mu)}$; $\tilde{q}=\frac{q}{E}\left(\frac{R}{h}\right)^2$; $U=\frac{\tilde{m}^2}{t(\tilde{m}^2+n^2)^2}+\frac{t(\tilde{m}^2+n^2)^2}{12(1-\mu^2)\tilde{m}^2}$; J and J_{cr} are bending and twisting moments of stringer inertia; F is the cross-sectional stringer area; E is the elastic modulus of the material; k is the number of stringers; $2n$, m is the number of half-waves generated during buckling in the circumferential and meridional directions.

In expression (7), according to [30], the number of half-waves in the circumferential direction $2n$ must not be a multiple of the number of stringers ($2n \neq n_1 k$). For special cases of buckling (8)–(9), the ratio of the multiplicity of the number of half-waves in the circumferential direction to the number of stringers ($2n = n_1 k$) can be satisfied.

Of the three considered cases of buckling (7)–(9), there can occur a case at which the smallest value of critical stresses $\sigma_{cr} = E\eta \cdot t$.

Choosing stringers from a strip with thickness h_r (the stringer height H is determined in accordance with the characteristic value that ensures the local stringer stability) as reinforcing elements, we will look for the values of the skin thickness h, rib thickness h_r, and number of stringers k, ensuring that, for a given load N, the shell has the smallest possible volume, and, at the same time, its strength and stability conditions are satisfied.

Thus, the problem is reduced to finding the minimum of the function

$$V = (2\pi Rh + kF)L \tag{10}$$

when the strength condition (6) and the constraint condition for the critical buckling force are met

$$2\pi Rh^2\left(1+\frac{\lambda h_p^2}{2\pi Rh}\right) \geq N, \tag{11}$$

where $\lambda = \frac{H}{h_p}$.

By introducing the notation $x_1=h$; $x_2=h_p$; $x_3=k$; $x_4=n$; $x_5=m$ and substituting into formulas (6)–(8), we obtain the mixed-integer non-linear programming problem: to find the non-negative values x_1, x_2, x_3 for which the objective function would take the minimum value

$$\Phi = \min\left(2\pi Rx_1 + \lambda x_2^2 x_3\right)L \tag{12}$$

and, at the same time, the conditions

$$2\pi R\{x_1[p](1+\tilde{\gamma}) - Rq(1+\tilde{\gamma}) + \mu Rq\tilde{\gamma} \geq N\}, \tag{13}$$

$$2\pi E x_1^2(1+\tilde{\gamma}x_3) \geq N, \tag{14}$$

where $\tilde{\gamma} = \frac{\lambda x_2^2}{2\pi Rx_1}$.

Problem (12)–(14) can be solved in two ways.

The first way. Five pseudo-random numbers are generated on a computer, with the help of which five random values h, h_r, k, n and m (k, n and m can only be taken as integers) are played. Then, for the first three variables h, h_r, k, such n and m (integers) are found that would deliver the minimum value to the critical stresses for all three cases of buckling. The minimization of critical stresses, as well as the search for the optimal parameters of the shell, is carried out using the proportional algorithm of continuous coordinate-wise self-learning with forgetting [1]. After finding the minimum critical stresses, constraints (13) and (14) are verified for satisfaction. If the constraints are satisfied, then the value of the objective function is computed, the found point is stored, and the memory vector is computed for each of the coordinates. Then, from the found point, with account taken of the search history, a step is made again, and so on. Steps are made simultaneously in all coordinates. The procedure described mathematically can be represented as follows:

$$\min\Phi = f_1\{\min\sigma_{\mathrm{cr}} + f_2(n,m)\}.$$

The described approach allows solving the problem of optimal design of a reinforced cylindrical shell, but such a solution is associated with significant search losses.

The second way. The solution of the posed problem (12)–(22) is based on the assumption of the independence of the forms n and m from the values of the skin and reinforcement parameters. This assumption may turn out to be incorrect if the optimization process is carried out only by minimizing the shell volume without account taken of the minimization of critical stresses. Control computations showed that in this case the critical load is significantly excessive, and the solution obtained is not always optimal.

The difficulty in optimally designing this type of shells lies in the fact that the optimal (in terms of volume) shell must correspond to its stress-strain state, at which the critical stresses acquire the minimum value.

We will obtain the implementation of this approach, using the proportional algorithm of continuous coordinate-wise self-learning with forgetting [1].

This algorithm runs as follows. The search system makes a step only in the direction that is probabilistically favorable. Search direction should be understood as a probabilistic gradient. As it is known, such a probability is formed using the increment of the objective function. We will use this property of the algorithm and propose the search system to advance to the point corresponding to the minimum of the objective function (for example, the minimum volume of a shell), not only depending on the increment of the objective function, but also on the increment of the critical stress function. In this case, it is assumed that the critical stresses should decrease. In turn, the critical stresses are a function of not only skin reinforcement parameters, but also of wave formation parameters. As a result, self-learning will be carried out using the optimized parameters, and the search system will move in the parameter space so that the critical stresses are minimized. The implementation of the learning algorithm of the search algorithm can be carried out by correspondingly changing the memory parameter u_i using the following recurrent dependence:

$$u_i^{(N+1)} = \vartheta \cdot u_i^{(N)} - \nu \cdot \Delta x_i^{(N)} \Delta\Phi_N \Delta\sigma_{\mathrm{cr}}.$$

Here, ϑ is the forgetting rate parameter $(0 \le \vartheta \le 1)$; $\nu>0$ is the learning rate parameter.

The results of control computations of the numerical example are close to the results obtained by other authors [29, 31]. It should be noted that the second method can drastically reduce the time required to solve the problem.

To illustrate this, let us consider an example of the optimal design of a stringer-reinforced cylindrical shell of radius R=1 m and of length L=2 m, hingedly supported at its edges under the axial compressive load N=835 kN and internal pressure q=0.535·10^4 Pa. Its elastic modulus E=6.867·10^4 MPa; p=147 MPa; μ=0.3; λ=13. The shell is reinforced with strip-shaped stringers. The optimization parameters have the following constraints: $0.1 \le h \le 1.5$ mm; $1.0 \le h_r \le 2.5$ mm; $1 \le k \le 100$; $1 \le n \le 50$; $1 \le m \le 50$.

The problem was solved in the second way with the use of a standard program for obtaining pseudorandom numbers uniformly distributed over a segment [0, 1]. The obtained values of the optimal parameters were compared with the data given in [29], where the same problem was considered, but the studies were carried out by the approximate solution of the corresponding direct problem. Table 1 shows the values of the optimal parameters, the volume and critical-stress parameters for all three forms of buckling, as well as the values of the limiting critical forces N_{cr} and of the bearing capacity N_{brg} from the strength condition. The lowest row of the same table shows the values of the optimal parameters and the optimal volume obtained in [1].

Table 1. Optimal design results for a compressed stringer-reinforced shell with account taken of three forms of buckling

$V_{\min}$,cm^3	h, mm	h_r, mm	k, pcs	η_1	η_2	η_3	N_{cr}, kN	N_{brg}, kN
12975	0.980	1.00	25	1.925	–	–	843.0	960.0
11334	0.880	1.00	9	–	2.426	–	836.0	889.0
17480	0.997	1.81	58	–	–	1.394	836.0	1293.0
16760	0.980	1.23	92	–	–	–	–	–

Table 2. Computed values of the optimal shell parameters and critical stress values for the three cases of buckling

Buckling cases	Cross-sectional areas, cm^2			Critical stress parameters	Critical stress value, MPa
	Shell	Shell wall	Stringers		
General	83.25	3.331	3.33	1.925	101.26
First special	67.94	55.250	1.41	2.426	123.05
Second special	89.21	62.610	26.59	1.394	93.71

Conclusions

Analyzing the numerical experiment results given in table 2, we can say the following:

1. The limiting constraints for all three forms of buckling were constraints on the critical force, but not on the bearing capacity of the shell. The safety factors for the bearing capacity for all three cases of buckling are greater than unity.

2. The most dangerous case of buckling is the second special case. The critical stresses for this case of buckling are smallest (σ_{cr}=93.71 MPa at the coefficient η=1.394).

3. For the second special case of buckling, the cross-sectional area of the shell was distributed practically in equal proportions between the shell wall and reinforcement elements. The primary-structure area was 42.5% of the total cross-sectional area of the shell. As a result, the thickness of the stringers increased and more stringers were required (58) compared to other cases of buckling.

4. Due to the fact that the second special case of buckling is most dangerous for the shell, the parameters corresponding to this case should be taken as optimal.

5. Two methods of organizing the search system for solving the problem of optimal design are proposed. The first method is traditional, requiring the inclusion of the integer waveform parameters into the optimization parameters.

The second method is based on such an organization of the search system, in which the advance to the extremum of the objective function depends not only on the increment of this function, but also on the increment of the critical stress function. In this case, it is assumed that the critical stresses should decrease. In turn, the critical stresses are a function of not only skin and reinforcement parameters, but also wave formation parameters. Therefore, learning is carried out using these parameters as well, and the search system will move in the parameter space so that the critical stresses are minimized. The use of this method significantly reduces search losses.

In conclusion, it should be noted that this work did not determine the degree of influence of the shell buckling (separately axial compression and internal pressure) on the value of the critical force, i.e. the value of load leading to a more dangerous case of buckling. In the future, such a study should be carried out.

As for the organization of the search procedure, the promising researches could be those connected with the creation of hybrid algorithms for finding extreme solutions. In particular, when combining methods of dynamic and random search, random search and genetic algorithms, one should apply reduction, i.e. reduce complexity to simplicity, and create so-called metaheuristic methods for seeking the global extremum, which are presented in [25]. Random search methods, with their undoubted advantages over regular methods for seeking extrema in the non-linear feasible space, have a significant drawback: they lack high accuracy. When reducing the feasible space, the accuracy of random search methods increases significantly.

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Оптимальне проектування підкріплених циліндричних оболонок при спільному осьовому стисканні та внутрішньому тиску

Г. В. Філатов

ДВНЗ Український державний хіміко-технологічний університет,
49005, Україна, м. Дніпро, пр. Гагаріна, 8

У статті розглядається застосування методу випадкового пошуку для оптимального проектування одношарових підкріплених ребрами жорсткості циліндричних оболонок при спільному осьовому стисканні і внутрішньому тиску з урахуванням пружно-пластичної роботи матеріалу. Як критерій оптимальності приймається мінімальний об'єм оболонки. Область пошуку оптимального розв'язку в просторі параметрів, що оптимізуються, обмежується умовами міцності і стійкості оболонки. Під час оцінки стійкості враховується дискретне розташування ребер. Крім умов міцності і стійкості оболонки, на область допустимих розв'язків накладаються обмеження на геометричні розміри параметрів, що оптимізуються. Складність при постановці задачі математичного програмування полягає в тому, що критичні напруження, які виникають в оптимальних стиснутих підкріплених циліндричних оболонках, є функцією не тільки параметрів обшивки і підкріплення, але й кількості напівхвиль в окружному та меридіональному напрямках, що утворюються в результаті втрати стійкості. У свою чергу, кількість цих напівхвиль залежить від варійованих параметрів оболонки. Отже, область пошуку стає нестаціонарною і при постановці задачі математичного програмування слід передбачати необхідність мінімізації функції критичних напружень за цілочисловими параметрами хвилеутворення на кожному кроці пошукової процедури. У зв'язку з цим пропонується методика розв'язання задачі оптимального проектування підкріплених сіткою ребер оболонок із застосуванням алгоритму випадкового пошуку, вивчення якого здійснюється не тільки в залежності від зменшення цільової функції, а й від збільшення критичних напружень на кожному кроці пошуку екстремуму. Метою роботи є демонстрація методики оптимізації такого роду оболонок, за якої використовується спеціальний алгоритм навчання системи пошуку, котра полягає в тому, що одночасно розв'язуються дві задачі математичного програмування: мінімізація вагової цільової функції і мінімізація критичних напружень. Методика, що пропонується, ілюструється на числовому прикладі.

Ключові слова: *підкріплена циліндрична оболонка, оптимальне проектування, випадковий пошук, критичні напруження втрати стійкості.*

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