DOI: https://doi.org/10.15407/pmach2021.03.052

UDC 624.04

AN INTEGRATED APPROACH TO THE OPTIMIZATION OF PLATES IN PLANE STRESS STATE OPERATED AT HIGH TEMPERATURES

Mark M. Fridman,
mark17@i.ua
ORCID: 0000-0003-3819-2776

Kryvyi Rih Metallurgical Institute of the National Metallurgical Academy of Ukraine
5, Stephan Tilha str.,
Kryvyi Rih, 5006, Ukraine

Many critical elements of building and machine-building structures during their operation are in difficult operating conditions (high temperature, aggressive environment, etc.). In this case, they can be subject to a double effect: corrosion and material damage. Corrosion leads to a decrease in the cross-section of a structure, resulting in stress increase therein. In turn, damage to the material is accompanied by the appearance of microcracks and voids therein, due to inelastic deformation (creep), leading to a deterioration in its physical properties (for example, the elastic modulus) and a sharp decrease in the stress values at which the structure is destroyed. This article continues the study in the field of the optimal design of structures subject to the aforementioned double effect by the example of the optimization of plates with holes in the plane stress state, exposed to high temperatures (in previous works, the use of this approach was demonstrated in the optimization of the bending elements of rectangular and I-sections). Used as a corrosion equation is the modified Dolinsky mode, which takes into account the (additional) effect of the protective properties of an anticorrosive coating on the corrosion kinetics. Taken as a kinetic equation describing the change in material damage, is Yu. N. Rabotnov's model, which enables to determine the duration of the incubation period of the beginning of the tangible process of material damage. To study the stress state of a plate, the finite element method is used. With a given contour of the plate, found is the optimal distribution of the thickness of the finite elements into which the given plate is divided. Acting as a constraint of the optimization problem is the parameter of damage to the plate material. The approach proposed in this work can be used to solve similar problems of the optimal design of structures operating under conditions of corrosion and material damage, using both analytical solutions and numerical methods.

Keywords: *corrosion, material damage, optimization.*

Introduction

Structures operated under certain conditions (high temperature, aggressive environment, etc.) can be subject to a double effect: corrosion and material damage. The first factor leads to a decrease in the section of a structure and, as a consequence, to a stress increase therein. As for the damage to the material: the appearance of microcracks and voids therein, due to inelastic strain (creep), it leads to a deterioration in physical properties (for example, the elastic modulus) and a sharp decrease in the stress values at which the structure is destroyed. To take into account the damage, L. M. Kachanov [1, 2] proposed a kinetic material damage model, which is characterized by a continuity parameter that changes from 1 in the initial state to 0 at the moment of destruction. In his work [3], Yu. N. Rabotnov uses a similar equation of material damage kinetics, where the value of damage ϖ is taken as a variable parameter, varying from 0 to 1. Other modifications of this model were performed in the works of Ya. Lemetri and Ya. L. Cheboshi [4, 5]. Using the principle of separability and introducing a stress-dependent normalized time parameter, the above models (in the case of one-dimensional tensile stresses) were modernized in the work of V. P. Golub [6]. A new approach to determining structural damage is illustrated by the example of static and cyclic loads. An original damage model was also proposed by L. A. Sosnovskiy and S. S. Shcherbakov [7]. The works [8, 9] were devoted to the review of research in this area.

The problem of the optimization of structures operated in the conditions of material damage are dealt with in the works of A. G. Kostyuk [10], M. I. Reitman [11], V. Prager [12], Yu. V. Nemirovsky [13], M. Zhichkovsky [14], etc.

In strength calculations, corrosion was taken into account in the works of Yu. G. Pronina and O. S. Sedova [15–19]. The optimization of structural elements under corrosion conditions is considered, in particular, in our works [20–25].

This article investigates the optimization of plates with holes in a plane stressed state, exposed to high temperatures, with a combined approach to taking into account corrosion and protective properties of an anti-corrosive coating (the use of this approach was demonstrated when optimizing bending elements in [22, 23]). It is assumed that perforated plates operate in hydrogen-containing media, and determined is the duration of the incubation period, after which a noticeable process of material damage begins (the double effect of corrosion and material damage was taken into account in the optimization of bending elements in [24, 25] using an analytical approach, unlike the numerical method used in this work). To study the stress state of a plate, the finite element method (FEM) is used. For a given contour of the plate, found is the optimal distribution of the thickness of the finite elements into which the given plate is divided.

Formulation of the Problem

Let us choose, as the basic corrosion equation, the model proposed by V. M. Dolinsky [26], which takes into account the effect of stresses on the corrosion wear of structures

$$\frac{dh_i}{dt}=\begin{cases} 0, \text{at } t<t_{ink} \\ -(\alpha+\beta\sigma_{ri}), \text{at } t\ge t_{ink} \end{cases} \tag{1}$$

where h_i, σ_{ri} are, respectively, the thickness and reduced stresses of the i-th finite element; α, β are constant (at a given temperature T) coefficients.

Currently, there are a number of mathematical models describing the decrease in the protective properties of coatings. Here is one of them [27]

$$\frac{dD_i}{dt}=-A(1+m\sigma_{ri}), \tag{2}$$

where D_i is a parameter characterizing the protective properties of the i-th finite element of the coating under consideration, the value of the parameter at the initial moment of time (t=0) being taken equal to one, and at the moment of loss of protective properties D_i=D_{ik}; A is the coefficient that takes into account both the effect of the type of protective coating and the nature of the aggressive environment; m is the coefficient that takes into account the effect of the stress state level on the kinetics of a decrease in the protective properties of the coating.

The following combined model of corrosive wear is proposed, with account taken of the decrease in the protective properties of the coating:

$$\frac{dh_i}{dt}=\begin{cases} -(\alpha+\beta\sigma_{ri})(1-D_i), \text{при } 0<D_i\le 1 \\ -2(\alpha+\beta\sigma_{ri}), \text{ при } D_i=0, \end{cases} \tag{3}$$

where the parameter D_i is derived from equation (2).

Taken as a kinetic equation describing the change in the damage in each element of the plate, ϖ_i, is the model of Yu. N. Rabotnov [3]

$$\frac{d\varpi_i}{dt}=\begin{cases} 0, \text{at } t<\tilde{t} \\ a[\sigma_{ri}/(1-\varpi_i)]^b, \text{at } t\ge\tilde{t}, \end{cases} \tag{4}$$

where a and b are constant (at a given temperature T) coefficients;

Since the values of the parameters a and b are constant at a given temperature, the following time approximations were taken:

$$a=a_0\exp(n\sqrt{t}),\ b=b_0-ct, \tag{5}$$

where n and c are constant coefficients; $\tilde{t}$ is the time after which a noticeable process of material damage begins.

The value of the last factor is determined by the formula [28]

$$\tilde{t}=a_t\exp(c/T)/P^{b_t},$$

where T is the specified operating temperature of the plate; P is the pressure of hydrogen-containing vapors; a_t, b_t, c are constant coefficients.

The reduced stresses that enter into (2), (3), and (4) are determined by the formula

$$\sigma_{ri}=\nu\sigma_1+(1-\nu)\sigma_e,$$

where σ_1 are maximum normal stresses; σ_e are effective von Mises stresses, ν is constant.

In our case

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{(\sigma_x - \sigma_y)^2/4 + \tau_{xy}^2}\,,\ \sigma_e = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2}\,. \quad (6)$$

Determination of the Stresses that Enter into (6)

At the initial time of their operation (t=0), the plates are heated to a high temperature T and are subjected to a load that creates a plane stress state therein. In this case, for a triangular FEM element in matrix form, we have the initial stresses

$$\{\boldsymbol{\sigma}\}_0 = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [\mathbf{D}][\mathbf{B}]\{\boldsymbol{\delta}\} - [\mathbf{D}]\{\boldsymbol{\varepsilon}\}\,,$$

where $[\mathbf{D}] = \frac{E_m}{1-\mu^2}\begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & \frac{1-\mu}{2} \end{bmatrix}$; $\{\boldsymbol{\varepsilon}_0\} = \begin{Bmatrix} \gamma T \\ \gamma T \\ 0 \end{Bmatrix}$; **[B]** is the matrix defining the geometric position of the finite element in the general coordinate system of the plate; E_m, μ, γ are, respectively, the averaged (over temperature) modulus of elasticity, Poisson's ratio, thermal expansion coefficient for the plate material.

In turn, the displacements $\{\boldsymbol{\delta}\}$ are determined from the expression

$$[\mathbf{K}]\{\boldsymbol{\delta}\} = \{\mathbf{F}\} + \{\mathbf{F}\}_T\,,$$

where $[\mathbf{K}] = [\mathbf{B}]^{\mathrm{T}}[\mathbf{D}][\mathbf{B}]h_i\Delta_i$ is the stiffness matrix of the i-th element; Δ_i is its area; $\{\mathbf{F}\}$ is the vector of nodal forces due to external load; $\{\mathbf{F}\}_T = [\mathbf{B}]^{\mathrm{T}}[\mathbf{D}]\{\boldsymbol{\varepsilon}_0\}h_i\Delta_i$ is the vector of nodal forces due to thermal expansion.

At $0<t<\tilde{t}$, the stresses in the plate (taking into account (2)) are determined as

$$\{\boldsymbol{\sigma}\} = [\mathbf{D}_1][\mathbf{B}]\{\boldsymbol{\delta}_1\} - [\mathbf{D}_1]\{\boldsymbol{\varepsilon}_p\} + \{\sigma\}_0\,, \quad (7)$$

where $[\mathbf{D}_1] = \frac{E_0}{1-\mu^2}\begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & \frac{1-\mu}{2} \end{bmatrix}$; E_0 is the modulus of elasticity at the temperature T; $\{\varepsilon_p\}$ are creep strains occurring at high temperatures.

Using the von Mises criterion, the creep strain rate is determined (as in [3]) as follows:

$$\varepsilon_x = \frac{\sigma_e^{p-1}}{a_p E_0}\left(\sigma_x - \frac{1}{2}\sigma_y\right), \qquad \varepsilon_y = \frac{\sigma_e^{p-1}}{a_p E_0}\left(\sigma_y - \frac{1}{2}\sigma_x\right), \quad (8)$$

where a_p and p are material constants.

In the case when $t \ge \tilde{t}$, the stresses in the plates with holes are determined by (7), the difference being that the elastic modulus that enters into (7) and (8) is determined by the formula

$$E = E_0(1+\lambda\varpi)^{-1},$$

where λ is the material constant.

Since the stresses in the plates are determined by the FEM, the solution of equations (1–4) is carried out by numerical integration, with a time step Δt.

Optimization Problem

Considering, as variable parameters, the thicknesses of the finite elements, i.e. $\overline{X} = \{x_1, x_2, ..., x_N\}^{\mathrm{T}} = \{h_1^0, h_2^0, ..., h_N^0\}^{\mathrm{T}}$ take, as the objective function, the volume of the plate at the initial moment of time $G = \sum_{i=1}^{N} \Delta_i x_i$.

Taking into account that the damage of each finite element of the plate should not exceed unity, the optimal design problem is reduced to finding the vector of minimum parameters $\overline{X}_{op}$, giving

$$G = \sum_{i=1}^{N} \Delta_i x_i \to \min$$

for $\varpi_i \le 1$, $i=1, 2, \ldots, N$.

This problem of nonlinear mathematical programming is solved using one of the efficient algorithms of the random search method [29].

Numerical Results

As a numerical illustration, consider the optimization of the perforated plate shown in figure 1.

Due to its symmetry, the fourth part of the plate was investigated by introducing restrictions on the axes of the symmetry. As shown by the preliminary analysis of the stress state of the given plate with different breakdown of the finite element mesh, an increase in the number of elements in the zone of stress concentrators (in the corners of the hole) does not allow obtaining a converging process, i.e. the stresses continue to increase. To smooth out this conflict, we will assume that the corners in the hole are rounded (this is not shown in figure 1), which makes it possible to reduce the stress concentration. As a result, the region of the given perforated plate is approximated by ten triangular finite elements.

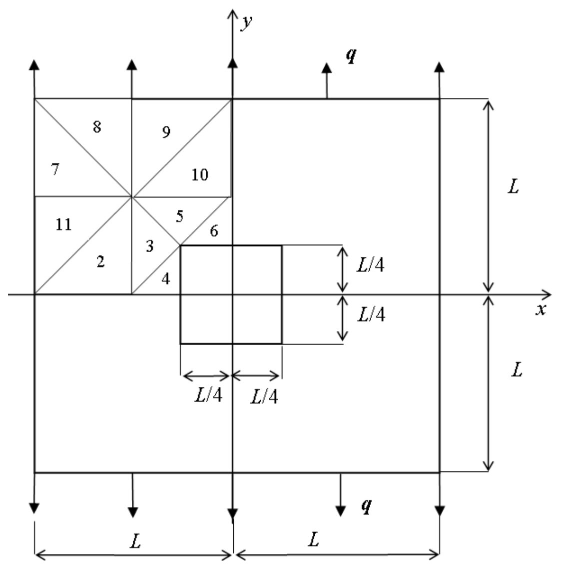

Fig. 1. Computational scheme of the perforated plate

The initial data of the problem are as follows: L=250 mm; α=0.1mm/year; $\beta=1\times10^{-3}$ mm/(MPa×year); t=10 years; Δt=0.2 year; $\gamma=1\times10^{-5}$ degrees^{-1}; P=100 MPa; a_p=1 year (MPa)$^{p-1}$; p=5; A=0.732 year^{-1}; m=0.005 MPa^{-1}; the material of the structure is carbon steel; λ=0,1; μ=0.3; ν=0.9; $a_t=1.48\times10^{-5}$ h (MPa)^{b_t}; b_t=1.73; c=13500 degrees^{-1}.

The following variants were considered for various loading values of the perforated plate: a) q_a=60 kN/m; b) $q_{\text{в}}$=90 kN/m; c) q_c=120 kN/m.

In addition, in each variant of the calculation (a, b, c), the values of the operating temperature T of the structure (at a constant load) varied: 1) T_1=500 °C; 2) T_2=510 °C; 3) T_3=520 °C. The corresponding values of the coefficients a and b (found from approximations (5), for which their boundary values at T_0=450 °C and T_f=550 °C were taken according to Odgvist [30]), as well as the values of the elastic moduli E_m and E_0 are as follows:

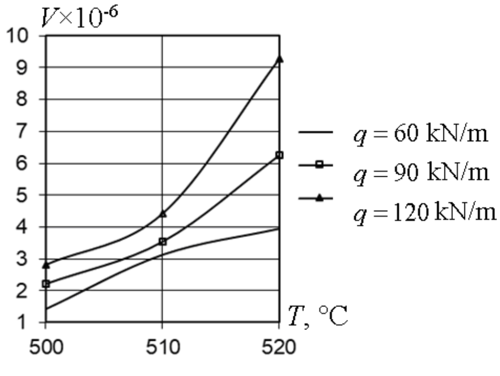

Fig. 2. Dependence of the perforated-plate volume on temperature

1) $a_1=19.49\times10^{-7}$ year^{-1}(MPa)$^{-b}$; b_1=2.25; $E_{m1}=1.825\times10^5$ MPa; $E_{01}=1.65\times10^5$ MPa;
2) $a_2=24.81\times10^{-7}$ year^{-1}(MPa)$^{-b}$; b_2=2; $E_{m2}=1.818\times10^5$ MPa; $E_{02}=1.636\times10^5$ MPa;
3) $a_3=30.98\times10^{-7}$ year^{-1}(MPa)$^{-b}$; b_3=1.75; $E_{m3}=1.811\times10^5$ MPa; $E_{03}=1.622\times10^5$ MPa.

The results of the numerical experiment are given in tables 1–3, and the graph of the dependence of the perforated-plate volume on temperature is shown in figure 2.

Table 1. Optimization results at T_1=500 °C

q, kN/m	Parameter	Finite element number i										G×10^{-3}, m^3
		1	2	3	4	5	6	7	8	9	10	
60	h_i^0, mm	6.00	5.00	2.05	22.00	12.00	2.80	6.00	6.00	4.00	6.00	1437
	h_i^T, mm	4.50	3.50	1.00	20.50	10.55	1.00	4.50	4.50	2.45	4.52	
	ϖ_i	0.383	0.336	0.330	0.425	0.285	0.028	0.383	0.778	0.960	0.360	
	σ_{ri}, MPa	5.20	5.02	5.00	5.32	4.80	1.97	5.20	5.80	5.83	5.12	
	t_{ink}, year	1.330	1.330	1.330	1.331	1.334	1.350	1.330	1.327	1.327	1.332	
90	h_i^0, mm	11.2	7.9	5.9	24.2	10.5	10.4	7.7	4.7	15.2	7.4	2219
	h_i^T, mm	9.60	6.50	4.47	22.40	9.46	8.77	6.47	3.56	13.55	5.45	
	ϖ_i	0.217	0.560	0.550	0.996	0.645	0.046	0.531	0.267	0.275	0.830	
	σ_{ri}, MPa	4.40	5.60	5.58	5.86	5.70	2.41	5.56	4.70	4.75	5.82	
	t_{ink}, year	1.337	1.329	1.329	1.327	1.334	1.350	1.330	1.340	1.340	1.327	
120	h_i^0, mm	10.9	5.0	12.3	41.4	20.4	3.0	11.1	2.8	16.7	15.8	2812
	h_i^T, mm	9.50	3.50	10.50	39.50	18.50	1.76	9.48	1.45	15.50	14.60	
	ϖ_i	0.530	0.510	0.514	0.700	0.725	0.052	0.517	0.950	0.674	0.170	
	σ_{ri}, MPa	5.56	5.52	5.53	5.75	5.79	2.54	5.53	5.85	5.73	4.05	
	t_{ink}, year	1.330	1.330	1.330	1.334	1.334	1.350	1.330	1.327	1.334	1.339	

Table 2. Optimization results at T_1=510 °C

q, kN/m	Parameter	Finite element number i										G×10^{-3}, m^3
		1	2	3	4	5	6	7	8	9	10	
60	h_i^0, mm	24.2	2.6	23.9	21.4	8.6	13.5	8.0	2.2	28.1	11.4	3125
	h_i^T, mm	22.90	1.00	22.70	19.60	7.66	11.80	6.65	1.00	26.80	9.77	
	ϖ_i	0.050	0.346	0.440	0.767	0.600	0.071	0.880	0.151	0.124	0.160	
	σ_{ri}, MPa	1.43	3.15	3.38	3.73	3.61	1.66	3.74	2.31	2.12	2.39	
	t_{ink}, year	1.356	1.345	1.321	1.341	1.342	1.356	1.341	1.351	1.352	1.331	
90	h_i^0, mm	13.40	27.30	4.18	36.50	13.00	27.30	12.10	18.10	7.06	11.80	3539
	h_i^T, mm	11.7	25.7	2.74	34.7	11.7	25.8	10.6	16.6	5.6	10.7	
	ϖ_i	0.380	0.240	0.232	0.347	0.412	0.081	0.634	0.775	0.956	0.576	
	σ_{ri}, MPa	3.24	2.78	2.75	3.16	3.32	1.76	3.65	3.73	3.78	3.53	
	t_{ink}, year	1.344	1.347	1.347	1.345	1.344	1.354	1.342	1.341	1.341	1.342	
120	h_i^0, mm	18.5	22.8	25.3	36.7	17.6	15.3	17.2	12.6	22.0	14.3	4422
	h_i^T, mm	16.7	21.7	23.7	35.6	16.65	13.8	15.7	11.7	20.6	12.6	
	ϖ_i	0.352	0.335	0.331	0.906	0.672	0.100	0.490	0.610	0.840	0.874	
	σ_{ri}, MPa	3.17	3.13	3.11	3.77	3.68	1.93	3.47	3.62	3.76	3.76	
	t_{ink}, year	1.344	1.345	1.345	1.341	1.341	1.353	1.343	1.341	1.341	1.341	

Table 3. Optimization results at T_1=520 °C

q, kN/m	Parameter	Finite element number i										G×10^{-3}, m^3
		1	2	3	4	5	6	7	8	9	10	
60	h_i^0, mm	13.00	7.80	26.60	42.70	20.40	14.90	14.30	4.42	36.80	12.10	3937
	h_i^T, mm	11.80	6.80	25.80	41.80	18.80	13.90	12.80	2.85	35.80	10.80	
	ϖ_i	0.948	0.811	0.767	0.788	0.766	0.080	0.644	0.280	0.294	0.834	
	σ_{ri}, MPa	2.160	2.136	2.120	2.130	2.120	0.862	2.060	1.561	1.600	2.140	
	t_{ink}, year	1.351	1.351	1.351	1.351	1.351	1.360	1.352	1.355	1.354	1.351	
90	h_i^0, mm	20.7	32.6	28.4	46.1	19.3	34.7	39.0	20.7	21.7	20.3	6242
	h_i^T, mm	19.8	31.8	26.8	44.8	17.8	33.9	37.9	19.8	20.8	18.8	
	ϖ_i	0.680	0.400	0.386	0.864	0.995	0.147	0.166	0.662	0.959	0.694	
	σ_{ri}, MPa	2.08	1.81	1.78	2.15	2.17	1.17	1.24	2.08	2.16	2.09	
	t_{ink}, year	1.352	1.354	1.354	1.351	1.351	1.358	1.358	1.352	1.351	1.352	
120	h_i^0, mm	45.8	29.4	51.0	66.5	25.9	52.5	35.5	50.4	6.8	62.0	9281
	h_i^T, mm	44.9	27.8	49.8	64.8	24.8	50.9	33.8	48.8	5.8	60.9	
	ϖ_i	0.235	0.415	0.424	0.992	0.872	0.138	0.413	0.537	0.980	0.117	
	σ_{ri}, MPa	1.46	1.83	1.85	2.17	2.15	1.13	1.83	1.98	2.16	1.04	
	t_{ink}, year	1.356	1.354	1.354	1.351	1.351	1.358	1.354	1.353	1.351	1.356	

Conclusions

The problem of optimal design of plane-stressed perforated plates, operated under conditions of corrosion and material damage, has been posed and solved with account taken of the protective properties of the anticorrosive coating.

It follows from the results obtained that an increase in the operating temperature of the perforated plate from 500 to 520 °C (as well as an increase in the load q applied to it, in the corresponding optimal versions leads to a sharp increase in all the thicknesses of the finite elements and, as a consequence, to an increase in the volume of the plate. This is particularly the case at T=520 °C and q=120 kN/m, where the volume of the plate increases several times.

Analyzing the graphs of the dependence of the volume of the perforated plate on temperature (Fig. 2), we can come to the conclusion that, for the given initial conditions, there is a power-law dependence of the type $V = f\left(T^{k(q)}\right)$ So, at q=60 kN/m (variant a), the coefficient $k(q)<1$ and at q=90 kN/m and above (variants b and c), $k(q)>1$, where $k(q)$ is some function depending on the load.

Note that with an increase in the operating temperature of the plate from 500 to 520 °C, the incubation period $\tilde{t}$ (at which the damage to the material can still be ignored) sharply decreases from 0.31 to 0.11 year. As for the time during which the end elements of the perforated plate completely lose their anticorrosive coating (D_i≈0), its value in this case ranges from 1.327 (at T=500 °C) to 1.358 (at T=520 °C), that is, it practically does not change.

In conclusion, it should be noted that the proposed approach to solving the problems of optimal design of structures operating under conditions of corrosion and material damage can be used to solve similar problems, using both analytical solutions and numerical methods.

References

1. Kachanov, L. M. (1974). *Osnovy mekhaniki razrusheniya* [Fundamentals of fracture mechanics]. Moscow: Nauka, 308 p. (in Russian).
2. Kachanov, L. M. (1985). *O vremeni razrusheniya v usloviyakh polzuchesti* [On the time of fracture under creep conditions]. *Izv. AN SSSR. Otd. tekhn. nauk – Proceedings of the USSR Academy of Sciences. Department of Technical Sciences*, no. 8, pp. 26–31 (in Russian).
3. Rabotnov, Yu. N. (1966). *Polzuchest elementov konstruktsiy* [Creep of structural elements]. Moscow: Nauka, 752 p. (in Russian).
4. Lemaitre, J. (1984). How to use damage mechanics. *Nuclear Engineering and Design*, vol. 80, iss. 2, pp. 233–245. https://doi.org/10.1016/0029-5493(84)90169-9.
5. Chaboche, J.-L. (1981). Continuous damage mechanics – a tool to describe phenomena before crack initiation. *Nuclear Engineering and Design*, vol. 64, iss. 2, pp. 233–247. https://doi.org/10.1016/0029-5493(81)90007-8.
6. Golub, V. P. (1996). Non-linear one-dimensional continuum damage theory. *International Journal of Mechanical Sciences*, vol. 38, iss. 10, pp. 1139–1150. https://doi.org/10.1016/0020-7403(95)00106-9.
7. Sosnovskiy, L. A. & Shcherbakov, S. S. (2011). *Kontseptsii povrezhdennosti materialov* [Concepts of material damage]. *Vestnik TNTU – Scientific journal of TNTU*, Special Issue (1), pp. 14–23 (in Russian).
8. Travin, V. Yu. (2014). *Otsenka povrezhdennosti materiala pri raschete prochnosti i dolgovechnosti elementov korpusnykh konstruktsiy* [Assessment of material damage in calculating the strength and durability of elements of hull structures]. *Izv. Tul. un-ta. Tekhn. nauki – Izvestiya Tula State University. Series: Technical science*, iss. 10, part 1, pp. 128–132 (in Russian).
9. Volegov, P. S., Gribov, D. S., & Trusov, P. V. (2017). Damage and fracture: Classical continuum theories. *Physical Mesomechanics*, vol. 20, iss. 2, pp. 157–173. https://doi.org/10.1134/S1029959917020060.
10. Kostyuk, A. G. (1953). *Opredeleniye profilya vrashchayushchegosya diska v usloviyakh polzuchesti* [Determination of the profile of a rotating disk under creep conditions]. *Prikl. matematika i mekhanika – Journal of Applied Mathematics and Mechanics*, vol. 17, iss. 5, pp. 615–618 (in Russian).
11. Reitman, M. I. (1967). Theory of the optimum design of plastics structures with allowance for the time factor. *Polymer Mechanics*, vol. 3, iss. 2, pp. 243–244. https://doi.org/10.1007/BF00858872.
12. Prager, W. (1968). Optimal structural design for given stiffness in stationary creep. *Journal of Applied Mathematics and Physics (ZAMP)*, vol. 19, iss. 2, pp. 252–256. https://doi.org/10.1007/BF01601470.
13. Nemirovskii, Yu. V. (1971). Design of optimum disks in relation to creep. *Strength of Materials*, vol. 3, iss. 8, pp. 891–894. https://doi.org/10.1007/BF01527642.

14. Zyczkowski, M. (1971). Optimal structural design in rheology. *Journal of Applied Mechanics*, vol. 38, iss. 1, pp. 39–46. https://doi.org/10.1115/1.3408764.
15. Pronina, Yu. & Sedova, O. (2021). Analytical solution for the lifetime of a spherical shell of arbitrary thickness under the pressure of corrosive environments: The effect of thermal and elastic stresses. *Journal of Applied Mechanics*, vol. 88, iss. 6, 061004. https://doi.org/10.1115/1.4050280.
16. Pronina, Yu., Maksimov, A., & Kachanov, M. (2020). Crack approaching a domain having the same elastic properties but different fracture toughness: Crack deflection vs penetration. *International Journal of Engineering Science*, vol. 156, 103374. https://doi.org/10.1016/j.ijengsci.2020.103374.
17. Pronina, Yu., Sedova, O., Grekov, M., & Sergeeva, T. (2018). On corrosion of a thin-walled spherical vessel under pressure. *International Journal of Engineering Science*, vol. 130, pp. 115–128. https://doi.org/10.1016/j.ijengsci.2018.05.004.
18. Pronina, Y. (2019). Design of pressurised pipes subjected to mechanochemical corrosion. In book: Advances in Engineering Materials, Structures and Systems: Innovations, Mechanics and Applications. London: Taylor & Francis, pp. 644–649. https://doi.org/10.1201/9780429426506-113.
19. Pronina, Y. G. (2017). An analytical solution for the mechanochemical growth of an elliptical hole in an elastic plane under a uniform remote load. *European Journal of Mechanics – A/Solids*, vol. 61, pp. 357–363. https://doi.org/10.1016/j.euromechsol.2016.10.009.
20. Pochtman, Yu. M. & Fridman, M. M. (1997). *Metody rascheta nadezhnosti i optimalnogo proyektirovaniya konstruktsiy, funktsioniruyushchikh v ekstremalnykh usloviyakh* [Methods for calculating the reliability and optimal design of structures operating in extreme conditions]. Dnepropetrovsk: Nauka i obrazovaniye, 134 p. (in Russian).
21. Fridman, M. M. & Elishakoff, I. (2020). Optimal thickness of a spherical shell subjected to double-sided corrosion. *International Journal of Sustainable Materials and Structural Systems (IJSMSS)*, vol. 4, no. 2/3/4, pp. 158–170. https://doi.org/10.1504/IJSMSS.2020.10031281.
22. Fridman, M. M. (2017). *Optimalnoye proyektirovaniye konstruktsiy pri kombinirovannom podkhode k uchetu korrozii i zashchitnykh svoystv antikorrozionnykh pokrytiy* [Optimal design of structures with a combined approach to accounting for corrosion and protective properties of anti-corrosion coatings]. *Problemy mashinostroyeniya – Journal of Mechanical Engineering – Problemy Mashynobuduvannia*, vol. 20, no. 3, pp. 64–68 (in Russian). https://doi.org/10.15407/pmach2017.03.064.
23. Fridman, M. (2018). Stepwise optimization of I-section flexible elements under a fuzzy approach to taking into account corrosion and protective properties of anticorrosive coating. *Journal of Mechanical Engineering – Problemy Mashynobuduvannia*, vol. 21, no. 3, pp. 58–64. https://doi.org/10.15407/pmach2018.03.058.
24. Fridman, M. M. (2019). Optimal Design of Bending Elements in Conditions of Corrosion and Material Damage. *Journal of Mechanical Engineering – Problemy Mashynobuduvannia*, vol. 22, no. 3, pp. 63–69. https://doi.org/10.15407/pmach2019.03.063.
25. Fridman, M. M. (2020). Optimization of Bendable I-Section Elements Subject to Corrosion and Material Damage. *Journal of Mechanical Engineering – Problemy Mashynobuduvannia*, vol. 23, no. 3, pp. 60–67. https://doi.org/10.15407/pmach2020.03.060.
26. Dolinskii, V. M. (1967). Calculations on loaded tubes exposed to corrosion. *Chemical and Petroleum Engineering*, vol. 3, iss. 2, pp. 96–97. https://doi.org/10.1007/BF01150056.
27. Karyakina, M. I. (1980). *Fiziko-khimicheskiye osnovy protsessov formirovaniya i stareniya pokrytiy* [Physical and chemical foundations of the formation and aging of coatings]. Moscow: Khimiya, 198 p. (in Russian).
28. Ovchinnikov, I. G. & Pochtman, Yu. M. (1995). *Tonkostennyye konstruktsii v usloviyakh korrozionnogo iznosa: raschet i optimizatsiya* [Thin-walled structures in conditions of corrosive wear: Calculation and optimization]. Dnepropetrovsk: Dnepropetrovsk University, 190 p. (in Russian).
29. Gurvich, I. B., Zakharchenko, B. G., & Pochtman, Yu. M. (1979). Randomized algorithm to solve problems of nonlinear programming. *Izvestiya Akademii nauk SSSR. Tekhnicheskaya kibernetika – Bulletin of the USSR Academy of Sciences. Engineering Cybernetics*, no. 5, pp. 15–17 (in Russian).
30. Odgvist, F. K. G. (1966). Mathematical theory of creep and creep rupture. Oxford Math. Mon., Clarendon Press, 234 p.

Received 04 June 2021

Комплексний підхід при оптимізації пластин в плоскому напруженому стані, що експлуатуються в умовах високої температури

М. М. Фрідман

Криворізький металургійний інститут Національної металургійної академії України,
50006, Україна, Дніпропетровська обл., м. Кривий Ріг, вул. Степана Тільги, 5

Багато відповідальних елементів будівельних і машинобудівних конструкцій під час своєї експлуатації перебувають в складних умовах роботи (висока температура, агресивне середовище і т.д.). У цьому випадку вони можуть бути схильними до подвійного ефекту: корозії і пошкодження матеріалу. Корозія призводить до зменшення перерізу конструкції, внаслідок чого в ній збільшуються напруження. У свою чергу пошкодженість матеріалу супроводжується появою в ньому мікротріщин і порожнеч, в результаті непружної деформації (повзучості), що призводить до погіршення його фізичних характеристик (наприклад модуля пружності) і різкого зниження величин напружень, за яких відбувається руйнування конструкції. У даній статті продовжено дослідження в області оптимального проектування конструкцій, схильних до подвійного ефекту: корозії і пошкодження матеріалу на прикладі оптимізації пластин з отворами, що знаходяться в плоскому напруженому стані і зазнають високої температури (в попередніх роботах використання такого підходу було продемонстровано при оптимізації згинальних елементів прямокутного і двотаврового перерізів). Як рівняння корозії використовується модифікована модель Долинського, що враховує вплив (додатковий) захисних властивостей антикорозійного покриття на кінетику корозії. Як кінетичне рівняння, що описує зміну пошкодження матеріалу, приймається модель Ю. М. Работнова і визначається тривалість інкубаційного періоду початку відчутного процесу пошкодження матеріалу. Для дослідження напруженого стану пластини використовується метод скінченних елементів. При заданому контурі пластини знаходиться оптимальний розподіл товщини скінченних елементів, на які розбивається дана пластина. Як обмеження задачі оптимізації виступає параметр пошкодження матеріалу пластини. Запропонований в роботі підхід може бути використаний при розв'язанні аналогічних задач оптимального проектування конструкцій, що працюють в умовах корозії і пошкодження матеріалу, з використанням як аналітичних розв'язків, так і числових методів.

Ключові слова*: корозія, пошкодженість матеріалу, оптимізація*.

Література

1. Качанов Л. М. Основы механики разрушения. М.: Наука, 1974. 308 с.
2. Качанов Л. М. О времени разрушения в условиях ползучести. *Изв. АН СССР*. Отд. техн. наук. 1985. № 8. С. 26–31.
3. Работнов Ю. Н. Ползучесть элементов конструкций. М.: Наука, 1966. 752 с.
4. Lemaitre J. How to use damage mechanics. *Nucl. Eng. Design.* 1984. Vol. 80. Iss. 2. P. 233–245. https://doi.org/10.1016/0029-5493(84)90169-9.
5. Chaboche J.-L. Continuous damage mechanics – a tool describe phenomena before crack initiation. *Nucl. Eng. Design*. 1981. Vol. 64. Iss. 2. P. 233–247. https://doi.org/10.1016/0029-5493(81)90007-8.
6. Golub V. P. Non-linear one-dimensional continuum damage theory. *Int. J. Mech. Sci*. 1996. Vol. 38. Iss. 10. P. 1139–1150. https://doi.org/10.1016/0020-7403(95)00106-9.
7. Сосновский Л. А., Щербаков С. С. Концепции поврежденности материалов. *Вестн. ТНТУ*. 2011. Спец. вып. Ч. 1. С. 14–23.
8. Травин В. Ю. Оценка поврежденности материала при расчете прочности и долговечности элементов корпусных конструкций. *Изв. Тул. ун-та*. *Техн. науки*. 2014. Вып. 10. Ч. 1. С. 128–132.
9. Волегов П. С. Грибов Д. С., Трусов П. В. Поврежденность и разрушение: классические континуальные теории. *Физ. мезомеханика*. 2015. Т. 18. № 4. С. 68–86.
10. Костюк А. Г. Определение профиля вращающегося диска в условиях ползучести. *Прикл. математика и механика*. 1953. Т. 17. № 5. С. 615–618.
11. Рейтман М. И. Теория оптимального проектирования конструкций, сделанных из пластика, принимая во внимание фактор времени. *Механика полимеров*. 1967. Т. 3. № 2. С. 357–360.
12. Prager W. Optimal structural design for given stiffness in stationary creep. *J. Appl. Math. and Physics.* 1968. Vol. 19. Iss. 2. P. 252–256. https://doi.org/10.1007/BF01601470.
13. Немировский Ю. В. Задача оптимального проектирования дисков в условиях ползучести. *Пробл. прочности.* 1971. № 8. С. 11–13.
14. Zyczkowski M. Optimal structural design in rheology. *J. Appl. Mech.* 1971. Vol. 38. Iss. 1. P. 39–46. https://doi.org/10.1115/1.3408764.

15. Pronina Yu., Sedova O. Analytical solution for the lifetime of a spherical shell of arbitrary thickness under the pressure of corrosive environments: The effect of thermal and elastic stresses. *J. Appl. Mech. ASME*. 2021. Vol. 88. Iss. 6. 061004. https://doi.org/10.1115/1.4050280.
16. Pronina Yu., Maksimov A., Kachanov M. Crack approaching a domain having the same elastic properties but different fracture toughness: Crack deflection vs penetration. *Intern. J. Eng. Sci. Elsevier*/ 2020. Vol. 156. 103374. https://doi.org/10.1016/j.ijengsci.2020.103374.
17. Pronina Yu., Sedova O., Grekov M., Sergeeva T. On corrosion of a thin-walled spherical vessel under pressure. *Intern. J. Eng. Sci. Elsevier*. 2018. Vol. 130. P. 115–128. https://doi.org/10.1016/j.ijengsci.2018.05.004.
18. Pronina Y. Design of pressurised pipes subjected to mechanochemical corrosion. In book: Advances in Engineering Materials, Structures and Systems: Innovations, Mechanics and Applications. London: Taylor & Francis, 2019. P. 644–649. https://doi.org/10.1201/9780429426506-113.
19. Pronina Y. G. An analytical solution for the mechanochemical growth of an elliptical hole in an elastic plane under a uniform remote load. *Europ. J. Mech.* – A/Solids. 2017. Vol. 61. P. 357–363. https://doi.org/10.1016/j.euromechsol.2016.10.009.
20. Почтман Ю. М., Фридман М. М. Методы расчета надежности и оптимального проектирования конструкций, функционирующих в экстремальных условиях. Днепропетровск: Наука и образование, 1997. 134 с.
21. Fridman M. M., Elishakoff I. Optimal thickness of a spherical shell subjected to double-sided corrosion. *Intern. J. Sustainable Materials and Structural Systems*. 2020. Vol. 4. No. 2/3/4. P. 158–170. https://doi.org/10.1504/IJSMSS.2020.10031281.
22. Фридман М. М. Оптимальное проектирование конструкций при комбинированном подходе к учету коррозии и защитных свойств антикоррозионных покрытий. *Пробл. машиностроения*. 2017. Т. 20. № 3. С. 64–68. https://doi.org/10.15407/pmach2017.03.064.
23. Fridman M. Stepwise optimization of I-section flexible elements under a fuzzy approach to taking into account corrosion and protective properties of anticorrosive coating. *J. Mech. Eng. – Problemy Mashynobuduvannia*. 2018. Vol. 21. No. 3. P. 58–64. https://doi.org/10.15407/pmach2018.03.058.
24. Fridman M. M. Optimal Design of Bending Elements in Conditions of Corrosion and Material Damage. *J. Mech. Eng. – Problemy Mashynobuduvannia*. 2019. Vol. 22. No. 3. P. 63–69. https://doi.org/10.15407/pmach2019.03.063.
25. Fridman M. M. Optimization of Bendable I-Section Elements Subject to Corrosion and Material Damage. *J. Mech. Eng. – Problemy Mashynobuduvannia*. 2020. Vol. 23. No. 3. P. 60–67. https://doi.org/10.15407/pmach2020.03.060.
26. Долинский В. М. Расчет нагруженных труб, подверженных коррозии. *Хим. и нефт. машиностроение.* 1967. № 2. С. 21–30.
27. Карякина М. И. Физико-химические основы процессов формирования и старения покрытий. М.: Химия, 1980. 198 с.
28. Овчинников И. Г., Почтман Ю. М. Тонкостенные конструкции в условиях коррозионного износа: расчет и оптимизация. Днепропетровск: Днепропетр. ун-т, 1995. 190 с.
29. Гурвич Н. Б., Захарченко В. Г., Почтман Ю. М. Рандоминизированный алгоритм для решения задач нелинейного программирования. *Изв. АН СССР. Техн. кибернетика*. 1979. № 5. С. 15–17.
30. Odgvist F. K. G. Mathematical theory of creep and creep rupture. Oxford Math. Mon., Clarendon Press, 1966. 234 p.