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## CONTEMPORARY APPROACHES TO THE VIBRATION DIAGNOSTICS OF ROTATING SHAFTS

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*Rotating shafts are the most vulnerable part of steam turbines. They are subjected to a wide range of static and dynamic loading combined with high temperature. Different types of damage of rotating shafts in most cases are the result of this long-term combined loading. To avoid catastrophic failure of structural elements during operation of turbines, it is urgent to use reliable damage detection methods. A lot of investigations demonstrated that most appropriate as applied to the turbine shafts in operation are the vibration-based methods, in particular, the change of natural frequencies, the appearance of non-linear resonances, the change of vibrational spectrum and others. The paper contains the comparative analysis of most effective approaches to the vibration diagnostics of damage in rotating shafts of steam turbines.*

**Keywords:** *rotating shaft, vibration diagnostics of damage, natural frequencies, non-linear effects.*

### Introduction

Structural elements of steam turbines in operation are subjected to a wide range of static and dynamic loading combined with high temperature and corrosive environment. Under such conditions damage may originate in different structural elements of turbine. Long time operation of turbine leads to the accumulation of damage and finally – to the catastrophic failure [1]. Vibration detection of damage in rotating shafts of steam turbines in operation is practically the only way to prevent technical disaster.

Recently a great many vibration methods of damage detection were proposed. They are based on the change of natural frequencies, mode shapes, modal damping, and non-linear effects [1]. To estimate the efficiency of vibration diagnostics to reveal damage of tolerable size is not a simple problem. This estimation is carried out either numerically or experimentally.

In such a way, numerical and experimental researches with small-scale models of cracked rotating shafts revealed utmost sensitivity of vibration diagnostics [2, 3]. The researches with large-scale models confirmed the capability quite reliable damage detection in rotating shafts based on vibration diagnostics [4], however the sensitivity of damage detection was found insufficient in certain cases. The progress of in operation diagnostics of functioning steam turbines is ambiguous: in [5] the case of successful detection of crack in turbine shaft was reported, but the authors of [6] did not notice any change of vibrations characteristics of turbine shaft with a crack of critical size.

Such a contradictory conclusions on the efficiency of vibration diagnostics of damage in rotating shafts can be explained by different reasons. One of them is the effect of compliance of damaged structure on the relative change of vibration parameters [7]. Besides, in operation of steam turbines mechanical characteristics of materials are influenced by the temperature, which affects the vibration characteristics of turbine's structural elements [8].

The accuracy of cracked shafts models is determined by the representation of proper effect of crack on the shaft's compliance. This problem can be quite accurately solved with the fracture mechanics by means of the interrelation of strain energy release rate and Stress Intensity Factor (SIF) [9]. Such a model of crack ensures reliable prediction of natural frequencies for cracked shaft, as well as it can reveal the non-linear effects in case of the so called closing crack (crack which periodically opens and closes during vibrations) [1].

The aim of the work is to demonstrate shortly the genesis and, in more detail, the contemporary approaches to the vibration diagnostics of rotating shafts, and to estimate the perspectives of vibration diagnostics of damage in rotating shafts.

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### Modelling of Crack

To simplify the problem of cracked shafts vibrations initial models represented crack as an open one [10]. Open crack behaves like a slot in vibrations of shaft. The advantage of such a model was its simplicity, but it could not predict non-linear effects. Besides this model overestimated the change of natural frequencies [1]. To improve the modelling of fatigue crack, the so-called closing crack was proposed [11]. This model is much more accurate, but the problem of vibration became non-linear.

The modelling of rotating shaft dynamic behavior to predict the fatigue crack needs the determination of time-periodic function, which governs the time-varying stiffness of the cracked rotor. Thus, the step function was proposed to model crack as either open or closed one [12, 13]. The different idea is to present the behavior of closing crack as a function of rotation angle [13]:

$$f(t) = 0.5(1 \pm \cos \omega t). \quad (1)$$

According to [13], eq. (1) is more accurate in case of deep crack as compared with the step function.

Two principally different ways to predict the compliance of cracked area was developed. First way suggested that crack can be either completely open or completely closed. Second one suggested smooth change of cracked area compliance in the process of shaft's rotation.

In the first case the state of crack was determined by the sign of stress in the cracked area: for the closed crack, the sine was negative and for the open crack – positive [14, 15]. The abrupt change of a rotating shaft's compliance was also determined by the bending curvature in the cracked area [16].

In the second case the smooth change of cracked area compliance was determined based on fracture mechanics approach and supposing that the front of crack is

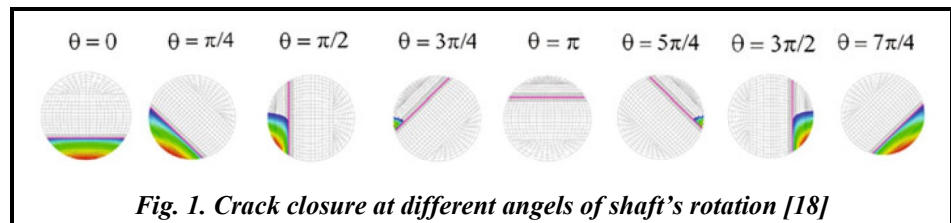


Fig. 1. Crack closure at different angles of shaft's rotation [18]

known for definite angles of rotation [17]. For instance, in [18] the change of compliance was calculated at different angles of rotation (Fig. 1).

The finite element models of cracked shafts, which can be found in the literature, vary significantly in the number of elements. However, as it follows from the analysis, in the problem of vibration diagnostics the number of elements is not as much important as the carefulness of the cracked area compliance determination.

### Initial Investigations on Vibration Diagnostics of Shafts

Initially vibration diagnostics of damage in rotating shafts was focused on their critical speed. The simplified model of shafts with an open crack evidently overestimated the change of critical speed as compared with a closing crack model. For instance, according to [10] open crack of relative depth  $a/D=0.3$  (where  $a$  is the depth of crack;  $D$  is the shaft's diameter) decreased critical speed of a shaft by 8.8%. Nevertheless, according to [19] the same depth but closing crack decreased critical speed by only 0.8%. In case of much deeper closing crack ( $a/D=0.5$ ) the decrease of critical speed did not exceed 2.5% [20].

As it was shown in numerous investigations [10, 11, 19, 20], the change of natural frequencies of shafts caused by transverse or slant crack in most cases is insufficient for reliable diagnostics of damage.

Because of this a lot of studies were directed to the development of much more sensitive methods of vibration diagnostics of damage in rotating shafts. Most of them are based on the non-linear effects [1]. Among these non-linear effects should be mentioned non-linear resonances, the non-linear distortion of stationary and transient vibrations, complex in plane trajectory of shaft's axis, coupling of longitudinal, bending, and torsional vibrations etc. At this the improvement of sensitivity and reliability of non-linear vibration diagnostics may be achieved by the variation of acceleration and deceleration on the stages of turbines start and stop.

### Non-linear Resonances of Rotating Shafts

Starting from Gasch [11] a lot of investigators discovered distinct increase of shaft's amplitude of vibration at speed of rotation close to  $1/2$ ,  $1/3$  etc. of the shaft first natural frequency with the increase of closing crack [4, 20, 22]. Fig. 2 demonstrates the non-linear resonances acceleration (forward  $f$ ) and deceleration (backward  $b$ ) of differently damaged rotating shaft. Close to  $1/2$  and  $1/3$  of the first critical speed one can observe noticeable

amplitudes of vibration. It is necessary to note, that all these results were derived with the use of small-scale models of shaft [1].

The investigation with piecewise linear model of a cracked rotating shaft [22] revealed its substantially non-linear behavior. In particular, variety of non-linear resonances manifested itself: superharmonic resonances of order 1/2 and 1/3, subharmonic resonance of order 2, supersubharmonic resonance of order 2/3, and a combination resonance ( $\omega_f + \omega_b$ ). Besides, the cracked shaft angular position of imbalance considerably affects the amplitudes of superharmonic resonances:

in the range of imbalance angular position  $90\text{--}270^\circ$  the amplitude of superharmonic resonance of order 1/2 can fall up to one order of magnitude.

According to investigations [15, 23] the non-linear effects in vibrations of large-scale shafts by mass of about several tons do not quite clear manifest itself, even in case of deep crack ( $a/D=0.5$ ) and low damping.

Opposite conclusion was made in [24] based on the study of cracked massive steel shaft model of about eight tones with one disk. In the range of rotational speed from 100 to 3600 rpm, the distinct superharmonic resonance of order 1/2 was revealed in case of quite moderate crack ( $a/D=0.25$ ).

Such a contradiction in conclusion of different studies can be explained by the effect of damping on the non-linear effects [1]. In the case of high damping capacity of cracked shaft, the non-linear resonances can be completely suppressed and, consequently, cannot be observed. On the contrary, small damping level is crucial to ensure that vibration diagnostics, based on non-linear effects, would be sufficiently sensitive [25].

### Spectral Analysis of Rotating Shafts Vibration

Spectral analysis of rotating shafts vibration around non-linear resonances distinctly reveals higher harmonics the amplitudes of which are related on the crack size [1]. Thus, the change of spectrum of vibration on the stages of start-up, shut-down and steady-state operation can be used as sensitive indicator of fatigue crack. On the stage of steady-state operation, the accounting for relative location of crack and imbalance is of importance [25].

Most sensitive to the presence of closing crack are first three harmonics of a shaft rotational frequency [25]. At this second harmonic is mostly affected, that is why it is considered as a primary indicator of crack [25, 26]. Besides, the sensitivity of second harmonic to the crack increases from the crack cross-coupling compliances [27]. The trend of third harmonic of vibration response of shaft is useful not only to detect the crack but also to identify is it either transverse or slant [28].

The change of vibrational spectrum on the stages of turbines start-up and shut-down is much more sensitive to damage than on the stage of steady-state operation [29]. At this the cosine acceleration scheme is more sensitive to damage than constant one regarding the second harmonic. Like the change of natural frequencies, the change of vibrational spectrum caused by crack decreases if crack locates close to bearings and couplings [1].

The appearance in the start-up bending vibration response of axial and torsional harmonics caused by the coupling also was suggested to apply for damage detection in rotating shafts [30].

Along with bending vibration response torsional vibration response can be used for vibration diagnostics of rotating shafts damage [31]. The excitation of torsional vibrations is caused by radial forces (for instance, imbalance or/and misalignment). The even harmonics of torsional vibration response spectrum are the indication of closing crack.

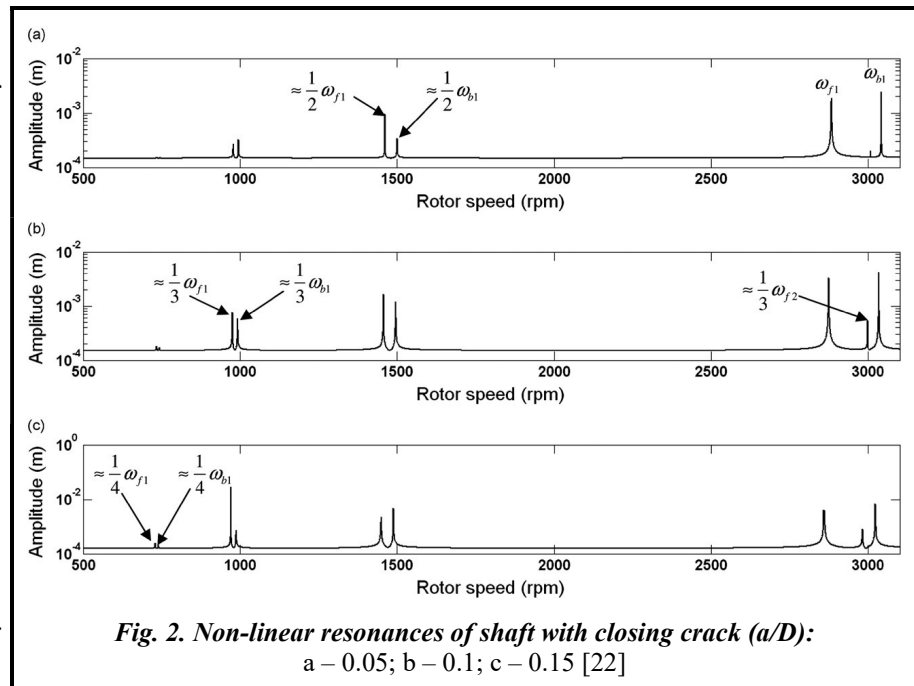


Fig. 2. Non-linear resonances of shaft with closing crack ( $a/D$ ):  
a – 0.05; b – 0.1; c – 0.15 [22]

Somewhat different idea on diagnostics of damage in shafts rotating in the transcritical and supercritical diapason of frequencies was formulated as follows: the diagnostics is based on the exposure of  $0.5\omega$ ,  $1.5\omega$  and  $4.5\omega$  components which, as it was shown in the research [32], are sensitive to closing crack in the range of critical frequencies from  $(2/3)\omega_{cr}$  to  $2\omega_{cr}$ .

Qualitatively slant crack influences the spectrum of torsional vibration response of cracked shaft similarly to transverse one on the spectrum of bending vibration response, but quantitatively – less intensive [33]. The intensity of torsional and bending vibration response of shaft with slant crack depends on its orientation [34]. The intensity increases while crack angle changes from  $30^\circ$  to  $60^\circ$ . In general, slant crack can be detected by the change of spectrum of torsional vibration response much easily than transverse one.

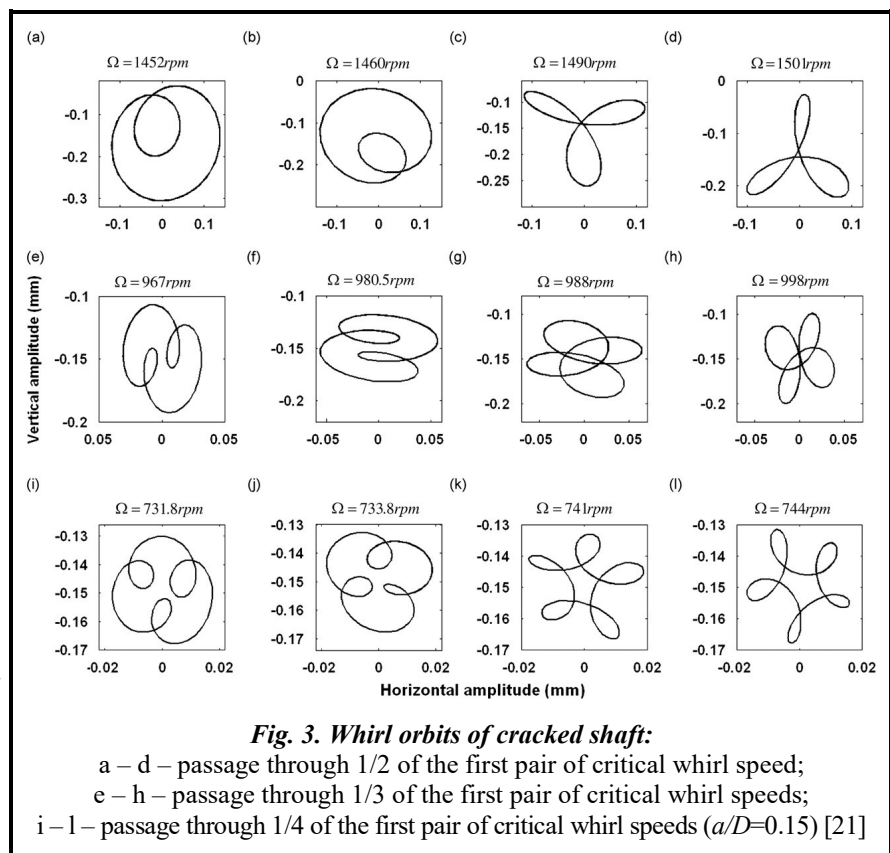
The spectrum of steady state vibration response of a shaft with slant crack is characterized by subharmonic frequency components [35].

The existence of coupling between longitudinal and bending vibration of shaft caused by crack is a prospective tool, which can be used along with the subcritical resonance, for the vibration diagnostics of damage in rotating shafts [17].

### Phase Trajectories of Rotating Shafts

The change phase trajectories also can be useful for damage detection in rotating shafts. In such a way, closing crack can be revealed by the two-loop and three-loop orbit motion of shaft's axis while passage through  $1/2$  and  $1/3$  of the critical speed, correspondingly [19, 25]. More general idea on the possible caused by crack distortion of the shaft's axis whirl orbits can be found in the research [21] (Fig. 3).

The presence of unstable range around critical speed, in which the transient whirl reversal motion and phase shift takes place, were demonstrated in a result of simulation [14]; if the imbalance is accounted for, the whirl reversal motion and phase shift vanish.



### Prospects of Vibration Diagnostics Improvement

To improve the reliability of damage detection in rotating shafts, there was proposed to apply intentional excitation of shafts in lateral, axial, or torsional directions.

Thus, short-term transient or harmonic torsional excitation and impulse axial excitation were applied to the rotating shaft with the aim to analyze the change of vibration response in lateral, axial and torsional directions caused by crack [36]. The proposed idea is based on the non-linear effect produced by closing crack, and on the coupling of bending and torsional vibrations. Besides, compressive axial impulses applied to the shaft produce the combination of measurable harmonics in the bending response spectrum.

To improve the reliability of vibration diagnostics of damage of shafts in steady-state operation the idea of long-term observation was proposed [4]. The idea is based on the permanent comparison of current vibration diagnostics parameters and reference values for intact structure. Among parameters under observation most sen-

sitive is the spectrum of vibration response close to non-linear resonances. A special so-called histogram technique was created to reveal minimal changes of spectrum caused by cracks of depth  $a/D=0.01\dots 0.02$  [37].

The sensitivity and reliability of vibration diagnostics is strongly dependent on the amount and location of sensors. The best position of sensors to ensure crack detection should be on the midspan rather than close to supports, as well as two or more sensors should be instrumented [24].

To improve the sensitivity of vibration diagnostics a lot of special treatment methods of vibration response were developed. Among them are wavelet transform, Hilbert transform, Wigner-Ville transformation, short time Fourier transform, genetic algorithms etc. [1].

### **Experimental Verification of Non-linear Effects**

Experimental verification of non-linear effects as applied to the vibration diagnostics of damage in rotating shafts were carried out on small- or large-scale models of shafts or on steam turbines in operation.

Laboratory tests on small-scale models of shafts demonstrated high sensitivity of non-linear effects to detect closing crack [19, 21]. Experiments corroborated the results of analytical and numerical investigations, according to which closing crack causes the appearance of second and higher harmonic in the vibration response spectrum and the change in the whirl orbit around the non-linear resonances. Tests on large-scale models confirmed the capability to detect closing crack by the appearance of higher harmonics in the vibration response spectrum [4, 37].

The practical use of non-linear vibration diagnostics as applied to steam turbines in operation was found to be controversial. The example of successful crack detection based on the appearance of subcritical resonances and higher harmonics in the spectrum of vibration response were reported in [38]. At the same time, a steam unit generator with quite a big crack ( $a/D=0.5$ ) did not demonstrated any sign of instability in different rotational speed [6].

### **Concluding Remarks**

The non-linear vibration diagnostics of damage in rotating shafts based on the non-linear resonances and on the spectrum of vibration at these resonances is perspective as far as it demonstrates a relatively high sensitivity to fatigue crack. At this the second harmonic in the vibration response around superharmonic resonance of order 2/1 could indicate the appearance of quite small cracks. The vibration responses during transitional regimes of rotating shafts operation, that is start-up and shut-down, are much more useful than the steady-state regime for the development of damage detection methods based on non-linear effects. Besides, the advantage of vibration diagnostics consists in the existence of natural dynamic loading, such as imbalance, which excites vibrations necessary for vibration diagnostics.

The main problem of damage detection in rotating turbine shafts consists in the inaccessibility of turbines structural elements for diagnostics. So, at a moment no other way exists to detect damage in operational turbine then to use the vibration diagnostics.

At the same time, it is necessary to note certain problems of vibration diagnostics based on the non-linear effects. First of all, the models of rotating shafts with closing crack are complicated and numerical solutions are time-consuming. In cases of substantial non-linearity, the solutions can be unstable. Besides, in such a problem it is necessary to have in mind that the non-linear damping and/or geometric non-linearity may cause the same non-linear effects as a closing crack. So, the presence of different types of non-linearity complicates the vibration diagnostics of damage based on the non-linear effects.

The evident drawback of non-linear effects as applied for damage detection is that they are dependent on the level of damping. High level of damping can fully suppress the non-linear effects caused by the closing crack making vibration diagnostics useless.

To improve the sensitivity and reliability of vibration diagnostics, based on the non-linear effects, different mathematical methods like wavelet transform, novelty detection and others can be used. The application of artificial impulse or periodic excitation in different directions to reveal specific features in vibration response of rotating shafts connected with a closing crack is another direction of future studies in the scope of non-linear vibration diagnostics.

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## Сучасні підходи до вібраційної діагностики обертових валів

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*Оберткові вали є найбільш вразливою частиною парових турбін. Вони зазнають широкого діапазону статичних і динамічних навантажень у поєднанні з високою температурою. Різні види пошкоджень обертових валів у більшості випадків є наслідком тривалого комбінованого навантаження. Щоб уникнути катастрофічного виходу з ладу елементів конструкції під час роботи турбін, необхідно використовувати надійні методи виявлення пошкоджень. Численні дослідження показали, що найбільш доцільними стосовно валів турбін при експлуатації є вібраційні методи, зокрема, зміна власних частот, поява нелінійних резонансів, зміна спектру*

коливань та інші. У статті проведено порівняльний аналіз найбільш ефективних підходів до вібродіагностики пошкоджень обертових валів парових турбін.

**Ключові слова:** обертовий вал, вібраційна діагностика пошкоджень, власні частоти, нелінійні ефекти.

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