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MULTICRITERIA PARAMETRIC OPTIMIZATION OF NONLINEAR ROBUST CONTROL WITH TWO DEGREES OF FREEDOM BY A DISCRETE-CONTINUOUS PLANT

[1] **Borys I. Kuznetsov**
kuznetsov.boris.i@gmail.com
ORCID: 0000-0002-1100-095X

[1] **Ihor V. Bovdui**
ibovduj@gmail.com
ORCID: 0000-0003-3508-9781

[1] **Olena V. Voloshko**
vinichenko.e.5@gmail.com
ORCID: 0000-0002-6931-998X

[2] **Tetyana B. Nikitina**
tatjana55555@gmail.com
ORCID: 0000-0002-9826-1123

[2] **Borys B. Kobylianskyi**
nnppiuipa@ukr.net
ORCID: 0000-0003-3226-5997

[1] Anatolii Pidhornyi Institute of Mechanical Engineering Problems of NAS of Ukraine,
2/10, Pozharskyi str., Kharkiv, 61046, Ukraine

[2] Educational scientific professional pedagogical Institute Ukrainian Engineering Pedagogical Academy,
9a, Nosakov str., Bakhmut, 84511, Ukraine

A multicriteria parametric optimization of nonlinear robust control with two degrees of freedom by a discrete-continuous plant has been developed to increase accuracy and reduce sensitivity to uncertain plant parameters. Such plants are mounted on a moving base, on which sensors for angles, angular velocities and angular accelerations are installed. To increase the accuracy of control, systems with two degrees of freedom, which include control with feedback and a closed-loop, and with direct connections and open-loop control of the setting and disturbing effects, are used. The multicriteria optimization of nonlinear robust control with two degrees of freedom by a discrete-continuous plant is reduced to the solution of the Hamilton-Jacobi-Isaacs equations. The robust control target vector is calculated as a solution of a zero-sum antagonistic vector game. The vector payoffs of this game are direct indexes performance vector presented in the system in different modes of its operation. The calculation of the vector payoffs of this game is related to the simulation of the synthesized nonlinear system for different operating modes of the system, input signals and values of the plant parameters. The solutions of this vector game are calculated on the basis of the system of Pareto-optimal solutions, taking into account the binary relations of preferences, on the basis of the stochastic metaheuristic of Archimedes optimization algorithm by several swarms. Thanks to the synthesis of nonlinear robust control with two degrees of freedom by a discrete-continuous object, it is shown that the use of synthesized controllers made it possible to increase the accuracy of control of an electromechanical system with distributed parameters of the mechanical part to reduce the time of transient processes by 1.5–2 times, reduce dispersion of errors by 1.3 times and reduce the sensitivity of the system to changes in the plant parameters in comparison with typical controllers used in existing systems. Further improvement of control accuracy is restrained by energy limitations of executive mechanisms and information limitations of measuring devices.

***Keywords**: discrete-continuous plant, nonlinear robust control, Hamilton-Jacobi-Isaacs equation, multicriteria parametric optimization, stochastic metaheuristic optimization algorithm.*

Introduction

Management of many technical plants and technological processes is carried out with the help of elongated constructions connecting the actuator with the working body. In particular, this applies to large antenna structures, solar arrays and spatially distributed antenna arrays, spaceships, crane booms, anthropomorphic robot arms, gun barrels, etc. During operation, torsional, longitudinal or transverse oscillations occur in such structures relative to the initial position. When controlling such an extended plant, it is necessary to take into account its own mechanical oscillations caused by the elastic properties of these extended control objects, and the control objects themselves are not perceived as electromechanical systems with distributed parameters of the mechanical part of the plant. At the same time, it was assumed that the mathematical model of the control object is known precisely. One of the central ideas of the modern theory of automatic control is the approach related to the control with the help of a single controller, not of one system with a clearly defined model, but of a whole class of systems, parameters, and possibly structures that can change during operation. This direction is currently developing intensively and has gained recognition as independent one, getting named as robust control.

The central problem of modern theory and practice of automatic control is the creation of systems capable of ensuring high accuracy of control under intense setting and disturbing effects of a wide range of frequencies [1–3]. The synthesis of control systems with two degrees of freedom, which combine the principles of open-loop and closed-loop control, allows, in a number of practical cases, to obtain an accuracy that is unattainable in feedback systems [4–5].

When working in different modes, various requirements are put forward to the multi-mass monitoring systems under development [6]. One of the main ones is the requirement for the stability of the synthesized system, that is, the ability of the system to support the technical requirements put forward to it when the parameters of the control object and external effects change within certain limits [7].

For such systems, in most practical cases, it is impossible to satisfy the technical requirements for the system with the help of typical proportional-integral-differential controllers, which necessitates the use of more complex controllers and modern methods of their synthesis [8]. One of the approaches to the problem of parametric synthesis is the optimization of system parameters, the structure of which is determined by solving the problem of optimal synthesis [9–10].

In the last decade, there have been developed methods to minimize the H_∞ is a norm, which serves as an effective indicator of the system's response to various types of actions in the presence of uncertainty in the description of the control object. In this regard, the development of a methodology for the synthesis of robust control systems for discrete-continuous objects for operation in the entire range of changes in the parameters of the control object with the help of one central controller designed for one set of parameters is an urgent task, which determined the direction of this research.

Such objects are installed on a moving base, on which sensors of angles, angular velocities and angular accelerations are mounted. To improve the accuracy of existing systems, control with two degrees of freedom is implemented, including closed-loop feedback control and closed-loop feedforward control by resistances and disturbances [9–10]. However, existing control systems use typical controllers, which limit the further improvement of the accuracy of such system.

The aim of the work is to develop a method of multicriteria parametric optimization of robust control with two degrees of freedom by a discrete-continuous plant to increase control accuracy and reduce sensitivity to plant parameters uncertainty.

Mathematical models of a discrete-continuous object

Let's consider the mathematical model of the object in the form of a rigid body and an elastic element, as shown in Fig. 1. The control of the object is driven by a synchronous motor with permanent magnets. On the basis of the sensors, the control of the frequency converter is created with the help of the controller to work on the given angular position or angular velocity of the object.

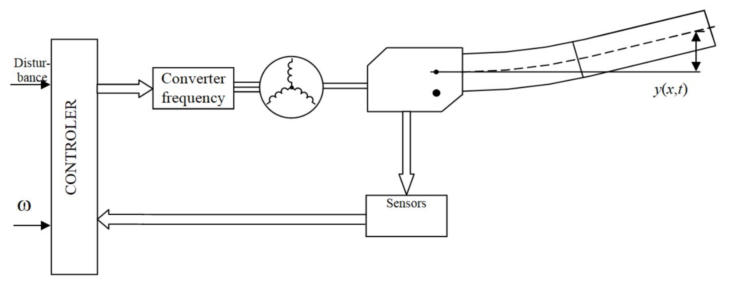

Fig. 1. Scheme of a discrete-continuous object

In addition to rotation around the axis, the plant performs elastic oscillations. Through $\varphi(t)$, we denote the angle of rotation of a rigid body in the inertial coordinate system, $y(x, t)$ is the deviation of points of the rod from the undeformed state. The control is carried out with the help of the moment $M_c(t)$, applied to the main rigid body. Disturbing moment $M_d(t)$ and the moment of friction $M_f(t)$ act relative to the axis of rotation of the rigid module, and $F_0(x, t)$ – the distributive force, which acts along the length of the elastic element. Then the equation of motion relative to the axis can be written in the following form [11–14]:

$$I_0 \frac{d^2\varphi(t)}{dt^2} - \int_r^{r+l} m_1(x) \frac{\partial^2 y(x,t)}{\partial t^2} dx = M_c(t) + M_d(t) - M_f(t). \tag{1}$$

This equation describes the free motion of a discrete-continuous plant, in which I_c is a characteristic of a discrete-continuous object as a rigid body, and $m_1(x)$ also characterizes the mutual effect of the movements of the rigid module and the oscillations of the elastic elements. Function $y(x, t)$ satisfies the equation of oscillations of an elastic beam

$$m_1(x)\frac{d^2\varphi(t)}{dt^2}+m(x)\frac{\partial^2 y(x,t)}{\partial t^2}+EI(x)\frac{\partial^4 y(x,t)}{\partial x^4}+\xi EI(x)\frac{\partial^5 y(x,t)}{\partial x^4 \partial t}=F_0(x,t), \tag{2}$$

where $EI(x)$ is the is the flexural rigidity of the plant; ξ is the internal damping coefficient of the plant material; $F_0(x, t)$ is the external disturbance, distributed along the length of the barrel, caused by vertical oscillations of the axis of the plant when the latter moves over rough terrain.

Let's introduce the function $y(x, t)$ in the form of the following decomposition

$$y(x,t)=\sum_{i=1}^{n}\gamma_i(x)T_i(t), \tag{3}$$

where n is the number of accounted forms of elastic oscillations of the plant. Then we obtain the following equations describing the motion of a discrete-continuous object under the action of control moment $M_c(t)$, disturbing moment $M_d(t)$ and turning friction moment $M_f(t)$, as well as the force distributed along the length of the plant $F_0(x, t)$, caused by vertical oscillations of the plant

$$I_0\frac{d^2\varphi(t)}{dt^2}-\sum_{i=1}^{n}\frac{d^2T_i(t)}{dt^2}\int_{r}^{r+l}m_1(x)\gamma_i(x)dx=M_c(t)+M_d(t)-M_f(t); \tag{4}$$

$$m_1(x)\frac{d^2\varphi(t)}{dt^2}+m(x)\sum_{i=1}^{n}\gamma_i(x)\frac{d^2T_i(t)}{dt^2}+EI(x)\sum_{i=1}^{n}\gamma_i^{1V}(x)T_i(t)+\xi EI(x)\sum_{i=1}^{n}\gamma_i^{1V}(x)\frac{dT_i(t)}{dt}=F(x,t). \tag{5}$$

Control accuracy is largely determined only by the first form of elastic oscillations. Considering only the first basic form of elastic oscillations, let's represent the function $y(x, t)$ in the form

$$y(x,t)=\gamma_0(x)T_0(t). \tag{6}$$

Then the dynamics equations (4)–(6) of the motion of a discrete-continuous object will take the form

$$I_0\frac{d^2\varphi(t)}{dt^2}-a_0\frac{d^2T_0(t)}{dt^2}=M_c(t)+M_d(t)-M_f(t); \tag{7}$$

$$a_0\frac{d^2\varphi(t)}{dt^2}+c_0\frac{d^2T_0(t)}{dt^2}+\xi b_0\frac{dT_0(t)}{dt}+b_0T_0(t)=f_0(t). \tag{8}$$

The disturbing moment is determined by the following ratio:

$$M_d=\mu_0\omega_d(t), \tag{9}$$

where μ_0 is the constant coefficient determined experimentally; $\omega_d(t)$ is the angular velocity of the plant in the vertical plane.

The second type of external disturbances is associated with accelerations of the plant relative to its vertical axis. These accelerations, being applied to the distributed masses of the plant, cause its elastic oscillations. The distributed forces applied to the tool and included in the equations of its bending oscillations are determined by the following relation

$$F_0(x,t)=m(s)\frac{d^2z(t)}{dt^2}, \tag{10}$$

where $m(x)$ is the mass per unit length of the plant; $\frac{d^2z(t)}{dt^2}$ is the acceleration of the object relative to its vertical axis.

Then we get

$$f_0(t)=\frac{d^2z(t)}{dt^2}\int_r^{r+l} m(x)\gamma_1(x)dx = k_z\frac{d^2z(t)}{dt^2}. \tag{11}$$

We will form random changes of the disturbing moment $M_d(t)$ and disturbing force $f_0(t)$ from uncorrelated sources of random signals, such as white noise of unit intensities, using shaping filters with transfer functions of oscillatory links.

$$W_d(p)=\frac{K_d\omega_{drR}^2}{p^2+2\xi_d\omega_{dr}p+\omega_{dr}^2}; \tag{12}$$

$$W_f(p)=\frac{K_f\omega_f^2}{p^2+2\xi_f\omega_f p+\omega_f^2}, \tag{13}$$

where ω_{dr}, ω_f are resonant frequencies of natural undamped oscillations; ξ_d, ξ_f are damping coefficients; K_d, K_f are gain coefficients of shaping filters.

The executive drive is a circuit of direct torque control of a synchronous motor, the equation of dynamics of which is written in the form

$$W_f(p)=\frac{K_m}{T^2{}_m p^2+2\xi_m T_m p+1}, \tag{14}$$

where T_m, ξ_m, K_m are time constant, damping coefficient and gain of the direct torque control circuit of the synchronous motor, respectively.

On the basis of the obtained equations (7)–(14), a scheme of the mathematical model of the plant with distributed parameters of the mechanical part is shown in Fig. 2.

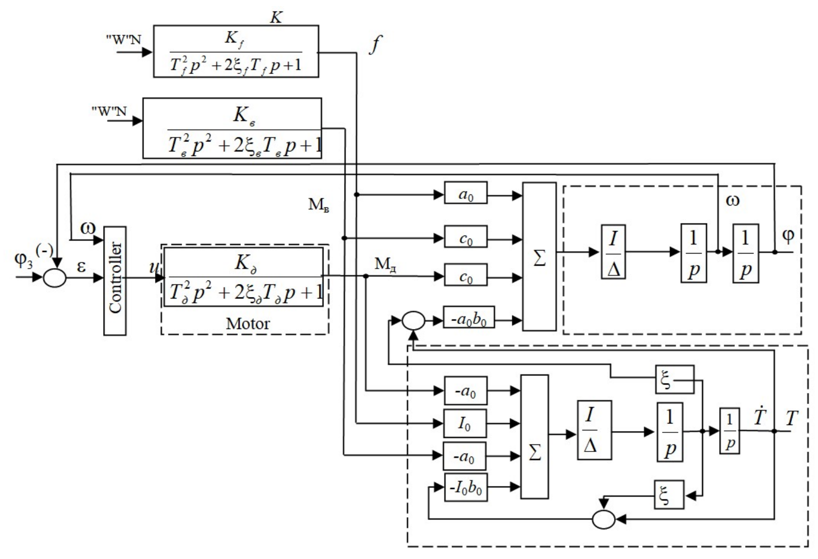

Fig. 2. Scheme of the mathematical model of the control system

Uncertainties of mathematical models of tracking systems

The central problem of the modern theory and practice of robust control is the design of systems capable of functioning effectively in conditions of uncertainty of the parameters of the plant model (1), and possibly also in the structure of the plant models, disturbing effects and measurement noises [1–3]. In the considered electromechanical tracking systems, the plant's moment of inertia changes most significantly during operation [4–5]. At the current stage of the development of the theory of automatic control systems, there is a need to search for such control in conditions of incomplete, unclear and imprecise knowledge of the characteristics of the control object and the environment in which this object functions, since the practice of designing and operating control systems objects showed that systems synthesized according to the criteria of modular and symmetric optima, as well as the quadratic quality criterion, are sensitive to changes in the parameters of the control object, characteristics of input and disturbing effects, to changes in the structure and parameters of the object model control, which is used in control loops [6–8]. Such systems may lose both optimality and performance if information about the object and operating environment is known with some degree of certainty or uncertainty [1].

Mathematical models of tracking systems

The accuracy improvement of electromechanical control systems is often held back by the imperfection of mechanical transmissions from the actuator to the working mechanism [1–3]. This manifests itself primarily when the bandwidth of the system increases, when the frequencies of the transmission's own mechanical oscillations, together with the drive and the working mechanism, fall into the range of operating frequencies of the control systems [4–5]. At the same time, it is necessary to take into account the presence of elastic elements between the shafts of the executive motor, gearbox and working mechanism, and instead of a single-mass model of the engine – working mechanism, – use two-, three-, and sometimes even multi-mass models [1]. The operating conditions of electromechanical systems are also complicated by the presence of a nonlinear dependence of the moment (force) of friction on the sliding speed of the working mechanism relative to the processed material. This dependence is often manifested in many operating modes of electromechanical systems at low (creeping) speeds of the working body movement. Moreover, for some mechanisms this mode is the one that works, and for others – emergency mode is the one that works [10].

The real kinematic scheme of electromechanical tracking systems contains elastic elements (elements of finite rigidity) [1–3]. The presence of elastic elements complicates the calculation scheme of the mechanical part of the system, turning it into a multi-mass one. Research has shown that with sufficient accuracy for practical calculations, the mechanical part of the system can be represented as a two-mass system.

Let's write down the electromechanical tracking system taking into account the models of executive motors and sensors as plant of the robust control system (1)–(14) with the state vector $x(t)$ in the standard form of the state equation

$$\frac{dx(t)}{dt} = f\big(x(t), u(t), \omega(t), \eta(t)\big), \tag{14}$$

where $u(t)$ is the control; $\omega(t)$ and $\eta(t)$ are vectors of the external signal and parametric disturbances; f is the nonlinear function.

The mathematical model (14) takes into account nonlinear frictional dependences on the shafts of the drive motor, rotating parts of the gearbox and plant, the gap between the teeth of the driving and driven gears, control restrictions, current, torque and motor rate, as well as the moment of inertia of the plant.

The measured output vector of the output system

$$y(t) = Y\big(x(t), \omega(t), u(t)\big) \tag{15}$$

is formed by various sensors that measured the angle, rate and acceleration of the plant [3].

Let's introduce the robust control target vector

$$z\big(x(t), u(t), \eta(t)\big) = Z\big(x(t), u(t), \eta(t)\big), \tag{16}$$

where Z is the nonlinear function.

We will form control $u(t)$ in the form of nonlinear feedbacks

$$u(t) = R\big(y(t)\big) \tag{17}$$

with a typical proportional-integral controller or by the full state vector of a nonlinear plant (14)

$$\frac{d\chi(t)}{dt} = G\big(\chi(t), u(t), \omega(t), y(t)\big), \tag{18}$$

which restores the full state vector $x(t)$ of the original plant (14) from the measured output vector $y(t)$ in (15).

Note that with the help of the observer (18) according to the measured output vector $y(t)$ of the original system (15), in addition to the state vector of the original object (15), the state variables of the vectors of controlling and disturbing effects on the system, as well as the vector of parametric effects, are also restored.

The solution to the problem of parametric design of an electromechanical tracking system are matrices $G=\{G_i\}$ and $R=\{R_i\}$, which are used to parameterize the controller's nonlinear functions R and the observer G in (17) and (18).

The situation becomes even more complicated when the presence of elastic elements is combined with the operation of the system in the falling section of the external friction characteristic, which can lead to the occurrence of sustained or even divergent mechanical oscillations [1–3].

Technical requirements for tracking systems

Different requirements are put forward to the operation of the control system in different modes [1–3]. As a rule, certain limitations are imposed on the quality of transient processes - the time of the first agreement, the time of regulation, over-regulation, etc. Of course, there is a limit to the maximum variance of the tracking or stabilization error when compensating for random disturbances, and constraints on the state and control variables must usually be met. Another requirement for control systems is to limit the errors of setting or compensating disturbing effects in the form of harmonic signals. One can set an input signal of one frequency or several characteristic operating frequencies, as well as a range of operating frequencies in which certain requirements must be met. Finally, for high-precision tracking systems, the characteristic mode of operation is the development of low speeds or small displacements. For this mode, smooth motion is usually specified in the form of appropriate criteria. The reason for the smooth motion of the working body at low speeds is the presence of nonlinearities of the dry friction type in drives, working bodies and elastic elements between the executive motor and the working body, which leads to jerky oscillations of the moving parts of the drive and the working body, which is accompanied by stops and disruptions of the moving parts in relation to the position of the stops.

Multipurpose parametric design

Usually, tracking systems are installed on a moving base. For such systems, there are separate measuring systems, which are used to measure setting and disturbing effects. The development of control systems with two degrees of freedom, which combine the principles of open-loop and closed-loop control, in a number of practical cases allows to obtain an accuracy that is unattainable in systems with feedback.

With this approach, information on supporting and disturbing effects is used to form control to obtain a minimum of errors in working out the system of interference and compensation of the interfering action. In this case, the conditions of invariance with regard to the determining and disturbing effects are reduced to the minimization of the norms of the transfer functions of the system error in accordance with the setting and disturbing effects.

To determine the robust controller with the target vector (16), the Hamiltonian function is considered

$$H\big(x(t),u(t),\eta(t)\big)=+V_x^T\big(x(t),u(t),\eta(t)\big)f\big(x(t),u(t),\eta(t)\big)+z^T\big(x(t),u(t),\eta(t)\big)-\frac{1}{\gamma^2}\eta^T(t)\eta(t)\,,$$

where V_x are partial derivatives with respect to the state vector $x(t)$ of the infinite-horizon performance functional (Lyapunov function); γ is the weighting coefficient that determines the degree of conservatism of the synthesized robust controller.

Necessary conditions for the extremum of the Hamiltonian function both in the control vector u, and in the external disturbance vector η are these equations

$$H_u\big(x(t),u^*(x(t)),\eta^*(x(t))\big)=0\,; \tag{19}$$

$$H_\eta\big(x(t),u^*(x(t)),\eta^*(x(t))\big)=0\,, \tag{20}$$

which are the Hamilton-Jacobi-Isaacs equations.

The solution of the system of Hamilton-Jacobi-Isaacs equations is carried out iteratively so that the first approximation is found in the form of solutions of two Riccati equations, which corresponds to the standard two Riccati approach to the synthesis of linear robust control. The synthesis of a nonlinear robust controller is reduced to the determination of nonlinear functions R and G by minimizing the norm of the target vector (16) by the control vector $u(t)$ and maximizing the same norm by the object uncertainty vector for the worst case. The nonlinear functions R and G are determined from the solutions of the Hamilton-Jacobi-Isaacs equations (19)–(20).

The dynamic characteristics of the synthesized system, which includes a nonlinear plant (14), that is closed by a nonlinear robust controller (17) and a nonlinear robust observer (18), are determined by the control system model of the system, the parameters of the measuring devices and the target vector (16). To correctly determine the target vector (16), we introduce a vector of unknown parameters, which are parametrization matrices of the nonlinear target vector function (16). We introduce multicriteria games with a game payoff vector – this is a direct index performance vector that is pre-sent to the system [1].

$$J(R,G,\eta)=\big[J_1(R,G,\eta),J_2(R,G,\eta),\dots,J_m(R,G,\eta)\big]^T\,, \tag{21}$$

in which the components $J_i(R, G, \eta)$ of the game payoff vector $J(R, G, \eta)$ are individual quality criteria imposed on the operation of the system in different modes of operation. The first player is the parameter vector of R and G controllers and the synthesized system, and its strategy is to minimize the payoff vector of the game. The second player is the vector of uncertainties η of the system plant model (1) and its strategy is to maximize the same payoff vector of the game.

The payoff vector of the game (21) is calculated by modeling the original nonlinear system (14), closed by the synthesized nonlinear controller (17)–(18) in different modes of operation with different input signals and for different values of system parameters. In such two degrees of freedom, the nonlinear robust control of the discrete-continuous closed-loop feedback of the plant is calculated on the basis of the state vector of the plant, and the direct control of the open-loop is calculated on the basis of the reference vector and the state vector of the disturbance models. In addition, nonlinear stable feedback and direct control are calculated simultaneously based on the solutions of the Hamilton-Jacobi-Isaacs equations (19)–(20).

From an engineering point of view, the problem of parametric optimization of controllers is of great practical importance, when the basic structure of the control system remains constant, and some vector parameters, and possibly the structure of the controller, change and thus counteract the change in the parameters of the external effects of the model (14) of the plant [1]. Using this approach, it is possible to synthesize quite good controllers that differ slightly from the optimal ones. However, their technical implementation can be significantly simplified. Such controllers have other common properties, for example, they are less sensitive (robust) to changes in parameters, structure of the plant and input signals.

The dynamic characteristics of the developed electromechanical tracking system, including a nonlinear object (14), which is closed by a robust proportional-differential controller (17) or a state controller (18), are determined by the control system model of the system (14), as well as the parameters of the measuring devices (15). Then the problem of multicriteria parametric design of a reliable electromechanical tracking system can be formulated in the form of vector game solutions (21).

Calculation of vector game solutions based on an optimization algorithm

Solutions of this vector game (21) from Pareto-optimal solutions [6] are calculated on the basis of the multi-swarm stochastic metaheuristic Archimedes optimization algorithm [15]. Currently, multi-agent methods of stochastic optimization, which use only the location speed of swarm particles, have become the most common. In fact, these algorithms are a first-order random search algorithm, since the search uses only the velocity of the particle – the first-order derivative of the scalar objective function or the gradient of the vector objective function [1]. When finding derivatives in deterministic methods, this algorithm is usually called gradient descent.

Recently, not only the speed, but also the acceleration of the change of the objective function has been used to increase the search speed. In this case, the acceleration of the swarm particles is found as changes in velocities at neighboring iterations.

Special non-linear algorithms of stochastic multi-agent optimization were used to find the solution of multicriteria games from Pareto-optimal solutions taking into account preference relations.

$$v_{ij}(t+1)=w_j v_{ij}(t)+c_{1j}r_{1j}(t)H(p_{1j}-\varepsilon_{1j}(t))[y_{ij}(t)-x_{ij}(t)]+c_{2j}r_{2j}(t)H(p_{2j}-\varepsilon_{2j}(t))[y_j^*(t)-x_{ij}(t)];$$

$$u_{ij}(t+1)=w_j u_{ij}(t)+c_{1j}r_{1j}(t)H(p_{1j}-\varepsilon_{1j}(t))[z_{ij}(t)-\delta_{ij}(t)]+c_{2j}r_{2j}(t)H(p_{2j}-\varepsilon_{2j}(t))[z_j^*(t)-\delta_{ij}(t)];$$

$$x_{ij}(t+1)=x_{ij}(t)+v_{ij}(t+1);$$

$$\delta_{ij}(t+1)=\delta_{ij}(t)+u_{ij}(t+1).$$

The use of the Archimedes algorithm [15] to solve the considered multicriteria parametric optimization made it possible to significantly reduce the time of calculating the solution of the vector game due to the fact that the calculation of the components of the game's payoff vector is related to the modeling of the system's operation in different operating modes and under various external effects, and therefore requires significant computing resources.

Results of computer simulation

Let's now consider the dynamic characteristics of the synthesized robust control systems of the electromechanical system with distributed parameters of the mechanical part of the plant. Fig. 3 shows the transient processes of plant state variables during processing by the mismatch $\Delta\varphi$=0.1, as well as the following plant state variables: a) the angle $\varphi(t)$ of deviation between the axis of the plant and the given direction, and b) its derivative $\frac{d\varphi(t)}{dt}$; c) the value of the function $T_0(t)$ and d) its derivative $\frac{dT_0(t)}{dt}$ in the representation of the function $y(x, t)$, which characterizes the deviation of the axis point of the plant from its state, which is not deformed; e) control moment $M_c(t)$ and f) its derivative $\frac{dM_c(t)}{dt}$.

Let's now consider the implementation of the state variables of the robust control system of the discrete-continuous control object under random disturbances. Fig. 4 shows the implementations of the following plant state variables: a) the angle $\varphi(t)$ of deviation between the axis of the object and the given direction, and b) its derivative $\frac{d\varphi(t)}{dt}$; c) the value of the function $T_0(t)$ and d) its derivative $\frac{dT_0(t)}{dt}$ in the representation of the function $y(x, t)$, which characterizes the deviation of the axis point of the plant from its state, which is not deformed; e) control moment $M_c(t)$ and f) its derivative $\frac{dM_c(t)}{dt}$.

A computer study of the dynamic characteristics of synthesized robust electromechanical control systems with distributed parameters of the mechanical part of the plant has been carried out. First of all, the effect of the parameters of the electromechanical system with distributed parameters of the mechanical part of the control object on the dynamic characteristics of robust control systems was analyzed. It was established that with the help of synthesized robust controllers for improved mathematical models, it is possible to reduce the time of transient processes by 1.5–2 times compared to a system with typical controllers. According to the results of the simulation of the synthesized system, it is shown that with the help of synthesized stochastic robust controllers, it was possible to increase the accuracy of controlling the electromechanical system with distributed parameters of the mechanical part and reduce the dispersion error by 1.3 times in comparison with the typical controllers used in existing systems, with random external effects. Further improvement of stabilization accuracy is restrained by energy limitations of executive mechanisms and information limitations of measuring devices.

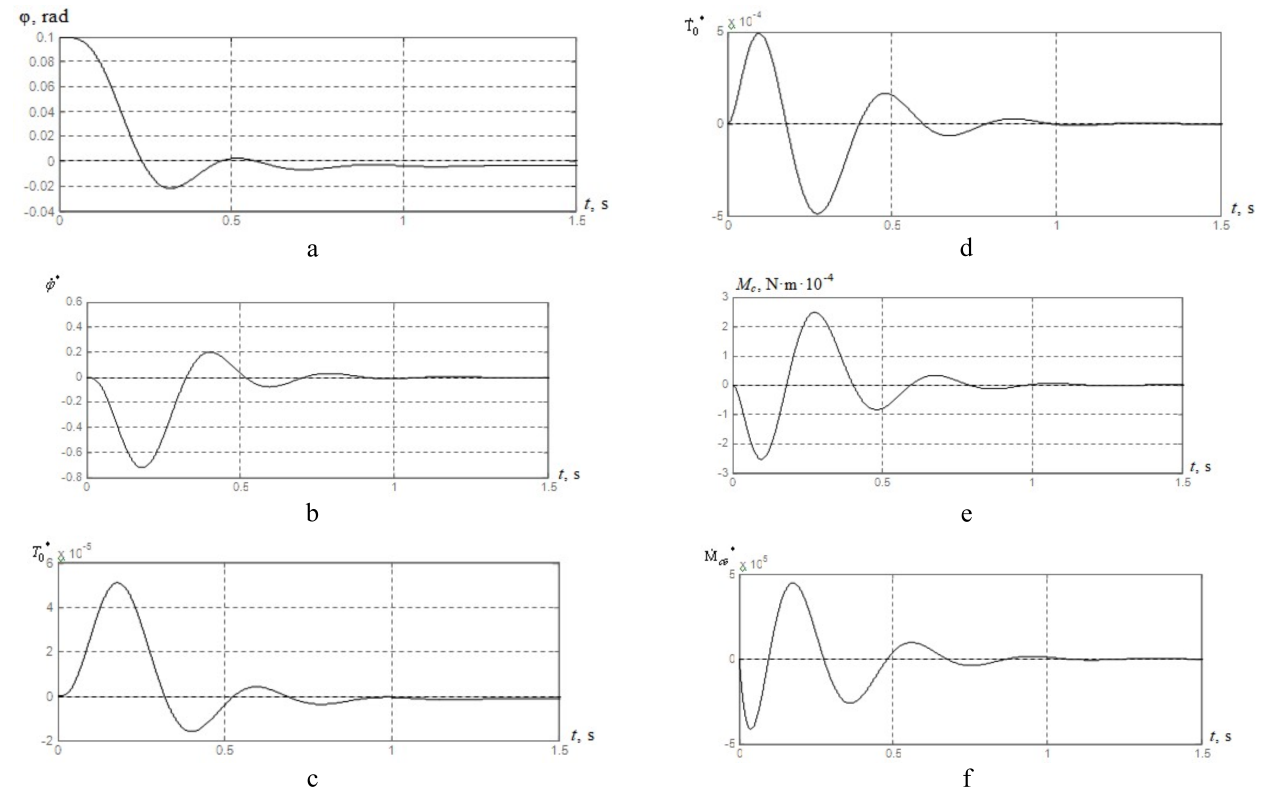

Fig. 3. Transient processes of plant state variables during processing by the mismatch system Δφ=0.1

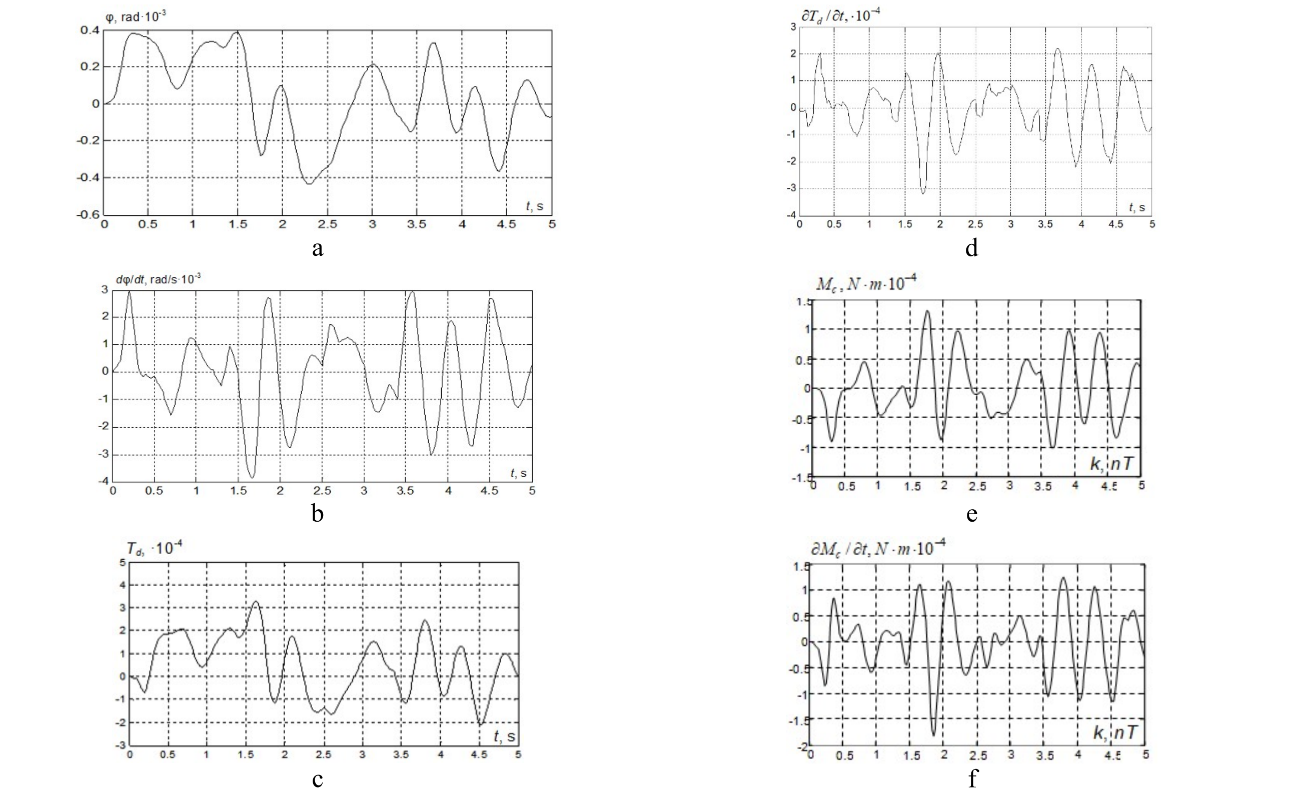

Fig. 4. Implementation of the state variables of the robust control system of the discrete-continuous control plant under random disturbances

Conclusions

1. The method of multicriteria parametric optimization of nonlinear robust control with two degrees of freedom by a discrete-continuous plant to increase accuracy and reduce sensitivity to uncertain parameters of the control plant has been developed.

2. The multicriteria synthesis of nonlinear robust control with two degrees of freedom by a discrete-continuous plant is reduced to the solution of the Hamilton-Jacobi-Isaacs equations. The robust control target vector is chosen as a solution of a zero-sum antagonistic vector game. The vector payoffs of this game are vectors of direct indicators of quality, which are the requirements for the operation of the system in different modes. The calculation of the vector payoffs of this game is related to the modeling of the synthesized nonlinear system for different operating modes of the system with different input signals and plant parameter values. The solutions of this vector game are calculated based on the multi-swarm stochastic metaheuristic Archimedes optimization algorithm.

3. As a result of the synthesis of nonlinear robust control with two degrees of freedom by a discrete-continuous object, it is shown that the use of synthesized controllers made it possible to increase control accuracy and reduce the sensitivity of the system to changes in the object parameters compared to existing systems.

References

1. Sushchenko, O., Averyanova, Yu., Ostroumov, I., Kuzmenko, N., Zaliskyi, M., Solomentsev, O., Kuznetsov, B., Nikitina, T., Havrylenko, O., Popov, A., Volosyuk, V., Shmatko, O., Ruzhentsev, N., Zhyla, S., Pavlikov, V., Dergachov, K., & Tserne, E. (2022). Algorithms for design of robust stabilization systems. In: Gervasi, O., Murgante, B., Hendrix, E. M. T., Taniar, D., Apduhan, B. O. (eds.) Computational Science and Its Applications – ICCSA 2022. Lecture Notes in Computer Science, vol. 13375. Cham: Springer, pp. 198–213. https://doi.org/10.1007/978-3-031-10522-7_15.
2. Shmatko, O., Volosyuk, V., Zhyla, S., Pavlikov, V., Ruzhentsev, N., Tserne, E., Popov, A., Ostroumov, I., Kuzmenko, N., Dergachov, K., Sushchenko, O., Averyanova, Yu., Zaliskyi, M., Solomentsev, O., Havrylenko, O., Kuznetsov, B., & Nikitina, T. (2021). Synthesis of the optimal algorithm and structure of contactless optical de-

vice for estimating the parameters of statistically uneven surfaces. *Radioelectronic and Computer Systems*, no. 4, pp. 199–213. https://doi.org/10.32620/reks.2021.4.16.

3. Volosyuk, V., Zhyla, S., Pavlikov, V., Ruzhentsev, N., Tserne, E., Popov, A., Shmatko, O., Dergachov, K., Havrylenko, O., Ostroumov, I., Kuzmenko, N., Sushchenko, O., Averyanova, Yu., Zaliskyi, M., Solomentsev, O., Kuznetsov, B., & Nikitina, T. (2022). Optimal method for polarization selection of stationary objects against the background of the Earth's surface. *International Journal of Electronics and Telecommunications*, vol. 68, no. 1, pp. 83–89. https://doi.org/10.24425/ijet.2022.139852.
4. Martynenko, G. (2020). Analytical method of the analysis of electromagnetic circuits of active magnetic bearings for searching energy and forces taking into account control law. Conference proceedings of the 2020 IEEE KhPI Week on Advanced Technology (IEEE KhPIWeek 2020), Ukraine, Kharkiv, October 5–10, 2020, pp. 86–91, https://doi.org/10.1109/KhPIWeek51551.2020.9250138.
5. Martynenko, G. & Martynenko, V. (2020). Rotor dynamics modeling for compressor and generator of the energy gas turbine unit with active magnetic bearings in operating modes. Proceedings of the 25th IEEE International Conference on Problems of Automated Electric Drive. Theory and Practice (IEEE PAEP 2020), Ukraine, Kremenchuk, September 21–25, 2020, pp. 1–4. https://doi.org/10.1109/PAEP49887.2020.9240781.
6. Ostroumov, I., Kuzmenko, N., Sushchenko, O., Pavlikov, V., Zhyla, S., Solomentsev, O., Zaliskyi, M., Averyanova, Yu., Tserne, E., Popov, A., Volosyuk, V., Ruzhentsev, N., Dergachov, K., Havrylenko, O., Kuznetsov, B., Nikitina, T., & Shmatko, O. (2021). Modelling and simulation of DME navigation global service volume. *Advances in Space Research*, vol. 68, iss. 8, pp. 3495–3507. https://doi.org/10.1016/j.asr.2021.06.027.
7. Averyanova, Yu., Sushchenko, O., Ostroumov, I., Kuzmenko, N., Zaliskyi, M., Solomentsev, O., Kuznetsov, B., Nikitina, T., Havrylenko, O., Popov, A., Volosyuk, V., Shmatko, O., Ruzhentsev, N., Zhyla, S., Pavlikov, V., Dergachov, K., & Tserne, E. (2021). UAS cyber security hazards analysis and approach to qualitative assessment. In: Shukla, S., Unal, A., Varghese Kureethara, J., Mishra, D. K., & Han, D. S. (eds) *Data Science and Security. Lecture Notes in Networks and Systems*, vol. 290, pp. 258–265. https://doi.org/10.1007/978-981-16-4486-3_28.
8. Zaliskyi, M., Solomentsev, O., Shcherbyna, O., Ostroumov, I., Sushchenko, O., Averyanova, Yu., Kuzmenko, N., Shmatko, O., Ruzhentsev, N., Popov, A., Zhyla, S., Volosyuk, V., Havrylenko, O., Pavlikov, V., Dergachov, K., Tserne, E., Nikitina, T., & Kuznetsov, B. (2021). Heteroskedasticity analysis during operational data processing of radio electronic systems. In: Shukla, S., Unal, A., Varghese Kureethara, J., Mishra, D. K., & Han, D. S. (eds.) *Data Science and Security. Lecture Notes in Networks and Systems*, vol. 290, pp. 168–175. https://doi.org/10.1007/978-981-16-4486-3_18.
9. Ostroumov, I., Kuzmenko, N., Sushchenko, O., Zaliskyi, M., Solomentsev, O., Averyanova, Yu., Zhyla, S., Pavlikov, V., Tserne, E., Volosyuk, V., Dergachov, K., Havrylenko, O., Shmatko, O., Popov, A., Ruzhentsev, N., Kuznetsov, B., & Nikitina, T. (2021). A probability estimation of aircraft departures and arrivals delays. In: Gervasi, O. et al. (eds.) *Computational Science and Its Applications (ICCSA 2021). Lecture Notes in Computer Science*, vol. 12950, pp. 363–377. https://doi.org/10.1007/978-3-030-86960-1_26.
10. Zhyla, S., Volosyuk, V., Pavlikov, V., Ruzhentsev, N., Tserne, E., Popov, A., Shmatko, O., Havrylenko, O., Kuzmenko, N., Dergachov, K., Averyanova, Yu., Sushchenko, O., Zaliskyi, M., Solomentsev, O., Ostroumov, I., Kuznetsov, B., & Nikitina, T. (2022). Statistical synthesis of aerospace radars structure with optimal spatio-temporal signal processing, extended observation area and high spatial resolution. *Radioelectronic and Computer Systems*, no. 1, pp. 178–194. https://doi.org/10.32620/reks.2022.1.14.
11. Maksymenko-Sheiko, K. V., Sheiko, T. I., Lisin, D. O., & Petrenko, N. D. (2022). Mathematical and computer modeling of the forms of multi-zone fuel elements with plates. *Journal of Mechanical Engineering – Problemy Mashynobuduvannia*, vol. 25, no. 4, pp. 31–38. https://doi.org/10.15407/pmach2022.04.032.
12. Hontarovskyi, P. P., Smetankina, N. V., Ugrimov, S. V., Garmash, N. H., & Melezhyk, I. I. (2022). Computational studies of the thermal stress state of multilayer glazing with electric heating. *Journal of Mechanical Engineering – Problemy Mashynobuduvannia*, vol. 25, no. 2, pp. 14–21. https://doi.org/10.15407/pmach2022.02.014.
13. Kostikov, A. O., Zevin, L. I., Krol, H. H., & Vorontsova, A. L. (2022). The optimal correcting the power value of a nuclear power plant power unit reactor in the event of equipment failures. *Journal of Mechanical Engineering – Problemy Mashynobuduvannia*, vol. 25, no. 3, pp. 40–45. https://doi.org/10.15407/pmach2022.03.040.
14. Rusanov, A. V., Subotin, V. N., Khoryev, O. M., Bykov, Yu. A., Korotaiev, P. O., & Ahibalov, Ye. S. (2022). Effect of 3D shape of pump-turbine runner blade on flow characteristics in turbine mode. *Journal of Mechanical Engineering – Problemy Mashynobuduvannia*, vol. 25, no. 4, pp. 6–14. https://doi.org/10.15407/pmach2022.04.006.
15. Hashim, F. A., Hussain, K., Houssein, E. H., Mabrouk, M. S., & Al-Atabany, W. (2021). Archimedes optimization algorithm: A new metaheuristic algorithm for solving optimization problems. *Applied Intelligence*, vol. 51, pp. 1531–1551. https://doi.org/10.1007/s10489-020-01893-z.

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Багатокритеріальна параметрична оптимізація робастного керування з двома ступенями свободи дискретно-континуальним об'єктом

[1] Б. І. Кузнецов, [1] І. В. Бовдуй, [1] О. В. Волошко, [2] Т. Б. Нікітіна, [2] Б. Б. Кобилянський

[1] Інститут проблем машинобудування ім. А. М. Підгорного НАН України
61046, Україна, м. Харків, вул. Пожарського, 2/10

[2] Навчально-науковий професійно-педагогічний інститут УІПА
84511, м. Бахмут, вул. Носакова, 9 а

Розроблено багатокритеріальну параметричну оптимізацію нелінійного робастного керування з двома ступенями свободи дискретно-континуальним об'єктом для підвищення точності і зниження чутливості до невизначених параметрів об'єкта. Такі об'єкти розміщені на рухомій основі, на якій встановлені датчики кутів, кутових швидкостей і кутових прискорень. Для підвищення точності керування використовуються системи із двома ступенями свободи, які включають керування із зворотним зв'язком і замкнутим контуром і із прямими зв'язками і розімкненими контурами керування по задаючому та збурюючому впливам. Багатокритеріальна оптимізація нелінійного робастного керування з двома ступенями свободи дискретно-континуальним об'єктом зведена до рішення рівнянь Гамільтона-Якобі-Айзекса. Вектор цілі робастного керування обчислюється у вигляді рішення антагоністичної векторної гри з нульовою сумою. Векторні виграші цієї гри – це прямі вимоги, які висуваються до системи в різних режимах її роботи. Розрахунок векторних виграшів цієї гри пов'язаний із моделюванням синтезованої нелінійної системи для різних режимів роботи системи, вхідних сигналів і значень параметрів об'єкта. Рішення цієї векторної гри розраховуються на основі системи Парето-оптимальних рішень з урахуванням бінарних відношень переваг на основі стохастичного метаевристичного алгоритму Архімеда оптимізації кількома роями. Завдяки синтезу нелінійного робастного керування з двома ступенями свободи дискретно-континуальним об'єктом показано, що використання синтезованих регуляторів дозволило підвищити точність керування електромеханічною системою з розподіленими параметрами механічної частини для зменшення часу перехідних процесів в 1,5–2 рази, зменшити дисперсію помилок в 1,3 рази і знизити чутливість системи до зміни параметрів об'єкта в порівнянні з типовими регуляторами, які використовуються в існуючих системах. Подальше підвищення точності керування стримується енергетичними обмеженнями виконавчих механізмів й інформаційними обмеженнями вимірювальних приладів.

Ключові слова: *дискретно-континуальний об'єкт, нелінійне робастне керування, рівняння Гамільтона-Якобі-Айзекса, багатокритеріальна параметрична оптимізація, стохастичний метаевристичний алгоритм оптимізації.*

Література

1. Sushchenko O., Averyanova Yu., Ostroumov I., Kuzmenko N., Zaliskyi M., Solomentsev O., Kuznetsov B., Nikitina T., Havrylenko O., Popov A., Volosyuk V., Shmatko, O., Ruzhentsev N., Zhyla S., Pavlikov V., Dergachov K., Tserne E. Algorithms for design of robust stabilization systems. In: Gervasi O., Murgante B., Hendrix E. M. T., Taniar D., Apduhan B. O. (eds) Computational Science and Its Applications – ICCSA 2022. Lecture Notes in Computer Science. Vol. 13375. Cham: Springer, 2022. P. 198–213. https://doi.org/10.1007/978-3-031-10522-7_15.
2. Shmatko O., Volosyuk V., Zhyla S., Pavlikov V., Ruzhentsev N., Tserne E., Popov A., Ostroumov I., Kuzmenko N., Dergachov K., Sushchenko O., Averyanova Yu., Zaliskyi M., Solomentsev O., Havrylenko O., Kuznetsov B., Nikitina T. Synthesis of the optimal algorithm and structure of contactless optical device for estimating the parameters of statistically uneven surfaces. *Radioelectronic and Computer Systems*. 2021. No. 4. P. 199–213. https://doi.org/10.32620/reks.2021.4.16.
3. Volosyuk V., Zhyla S., Pavlikov V., Ruzhentsev N., Tserne E., Popov A., Shmatko O., Dergachov K., Havrylenko O., Ostroumov I., Kuzmenko N., Sushchenko O., Averyanova Yu., Zaliskyi M., Solomentsev O., Kuznetsov B., Nikitina T. Optimal method for polarization selection of stationary objects against the background of the Earth's surface. *International Journal of Electronics and Telecommunications*. 2022. Vol. 68. No. 1. P. 83–89. https://doi.org/10.24425/ijet.2022.139852.
4. Martynenko G. Analytical method of the analysis of electromagnetic circuits of active magnetic bearings for searching energy and forces taking into account control law. Conference proceedings of the 2020 IEEE KhPI Week on Advanced Technology (IEEE KhPIWeek 2020), Ukraine, Kharkiv, October 5–10, 2020. P. 86–91, https://doi.org/10.1109/KhPIWeek51551.2020.9250138.
5. Martynenko G., Martynenko V. Rotor dynamics modeling for compressor and generator of the energy gas turbine unit with active magnetic bearings in operating modes. Proceedings of the 25th IEEE International Confer-

ence on Problems of Automated Electric Drive. Theory and Practice (IEEE PAEP 2020), Ukraine, Kremenchuk, September 21–25, 2020. P. 1–4. https://doi.org/10.1109/PAEP49887.2020.9240781.

6. Ostroumov I., Kuzmenko N., Sushchenko O., Pavlikov V., Zhyla S., Solomentsev O., Zaliskyi M., Averyanova Yu., Tserne E., Popov A., Volosyuk V., Ruzhentsev N., Dergachov K., Havrylenko O., Kuznetsov B., Nikitina T., Shmatko O. Modelling and simulation of DME navigation global service volume. *Advances in Space Research*. 2021. Vol. 68. Iss. 8. P. 3495–3507. https://doi.org/10.1016/j.asr.2021.06.027.
7. Averyanova Yu., Sushchenko O., Ostroumov I., Kuzmenko N., Zaliskyi M., Solomentsev O., Kuznetsov B., Nikitina T., Havrylenko O., Popov A., Volosyuk V., Shmatko O., Ruzhentsev N., Zhyla S., Pavlikov V., Dergachov K., Tserne E. UAS cyber security hazards analysis and approach to qualitative assessment. In: Shukla S., Unal A., Varghese Kureethara J., Mishra D. K., Han D. S. (eds.) *Data Science and Security. Lecture Notes in Networks and Systems*. 2021. Vol. 290. P. 258–265. https://doi.org/10.1007/978-981-16-4486-3_28.
8. Zaliskyi M., Solomentsev O., Shcherbyna O., Ostroumov I., Sushchenko O., Averyanova Yu., Kuzmenko N., Shmatko O., Ruzhentsev N., Popov A., Zhyla S., Volosyuk V., Havrylenko O., Pavlikov V., Dergachov K., Tserne E., Nikitina T., Kuznetsov B. Heteroskedasticity analysis during operational data processing of radio electronic systems. In: Shukla, S., Unal, A., Varghese Kureethara, J., Mishra, D. K., & Han, D. S. (eds) *Data Science and Security. Lecture Notes in Networks and Systems*. 2021. Vol. 290. P. 168–175. https://doi.org/10.1007/978-981-16-4486-3_18.
9. Ostroumov I., Kuzmenko N., Sushchenko O., Zaliskyi M., Solomentsev O., Averyanova Yu., Zhyla S., Pavlikov V., Tserne E., Volosyuk V., Dergachov K., Havrylenko O., Shmatko O., Popov A., Ruzhentsev N., Kuznetsov B., Nikitina T. A probability estimation of aircraft departures and arrivals delays. In: Gervasi, O. et al. (eds.) *Computational Science and Its Applications (ICCSA 2021). Lecture Notes in Computer Science*. 2021. Vol. 12950. P. 363–377. https://doi.org/10.1007/978-3-030-86960-1_26.
10. Zhyla S., Volosyuk V., Pavlikov V., Ruzhentsev N., Tserne E., Popov A., Shmatko O., Havrylenko O., Kuzmenko N., Dergachov K., Averyanova Yu., Sushchenko O., Zaliskyi M., Solomentsev O., Ostroumov I., Kuznetsov B., Nikitina T. Statistical synthesis of aerospace radars structure with optimal spatio-temporal signal processing, extended observation area and high spatial resolution. *Radioelectronic and Computer Systems*. 2022. No. 1. P. 178–194. https://doi.org/10.32620/reks.2022.1.14.
11. Maksymenko-Sheiko K. V., Sheiko T. I., Lisin D. O., Petrenko N. D. Mathematical and computer modeling of the forms of multi-zone fuel elements with plates. *Journal of Mechanical Engineering – Problemy Mashynobuduvannia*. 2022. Vol. 25. No. 4. P. 31–38. https://doi.org/10.15407/pmach2022.04.032.
12. Hontarovskyi P. P., Smetankina N. V., Ugrimov S. V., Garmash N. H., Melezhyk I. I. Computational studies of the thermal stress state of multilayer glazing with electric heating. *Journal of Mechanical Engineering – Problemy Mashynobuduvannia*. 2022. Vol. 25. No. 2. P. 14–21. https://doi.org/10.15407/pmach2022.02.014.
13. Kostikov A. O., Zevin L. I., Krol H. H., Vorontsova A. L. The optimal correcting the power value of a nuclear power plant power unit reactor in the event of equipment failures. *Journal of Mechanical Engineering – Problemy Mashynobuduvannia*. 2022. Vol. 25. No. 3. P. 40–45. https://doi.org/10.15407/pmach2022.03.040.
14. Rusanov A. V., Subotin V. N., Khoryev O. M., Bykov Yu. A., Korotaiev P. O., Ahibalov Ye. S. Effect of 3D shape of pump-turbine runner blade on flow characteristics in turbine mode. *Journal of Mechanical Engineering – Problemy Mashynobuduvannia*. 2022. Vol. 25. No. 4. P. 6–14. https://doi.org/10.15407/pmach2022.04.006.
15. Hashim F. A., Hussain K., Houssein E. H., Mabrouk M. S., Al-Atabany W. Archimedes optimization algorithm: A new metaheuristic algorithm for solving optimization problems. *Applied Intelligence*. 2021. Vol. 51. P. 1531–1551. https://doi.org/10.1007/s10489-020-01893-z.