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EFFECTIVE FORMALIZATION OF DESIGN PROCESSES AS A KEY FACTOR IN ACHIEVING OPTIMAL SOLUTIONS WHEN CREATING THE FINAL STAGES OF STEAM TURBINES

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Based on the existing experience in designing and constructing of the last stage blades of large (critical) length and the analysis of literary sources, the features of the methodology for formalizing the processes of creating such blades, taking into account their specific features (large radial dimensions, suboptimal relative grid steps $\bar{t}$ =0.25–1.0, high static and dynamic loads), are established. A parametric formalization of the main modeling dependencies of the processes on which the creation of rotor blades is based is given: the thermo-gas-dynamic process, blade design and the technological process of manufacturing. The need to create systems (subsystems) for automated design of blades of large length with the presence of a model of the technological process of blade manufacturing in the system is substantiated. It is based on the conclusions that even small deviations from the design option within the tolerance limits during blade manufacturing affect the thermo-gas-dynamic characteristics of the stage, especially when it comes to throat areas. A formalized probabilistic-statistical mathematical model that allows to describe the technological deviations of the blade surfaces taking into account the processing modes used in finish milling with a reliability satisfactory for practical calculations has been developed. This makes it possible to take into account the influence of manufacturing errors and specific features of machine equipment on the blade strength indicators, its gas-dynamic characteristics, and also on the efficiency of the stage operation at the design stage. A two-level approach to the design process, which allows using a two-dimensional model to conduct a directed search for the best solution in an automated mode, analyzing hundreds of options taking into account a wide range of constraints, is proposed. Subsequently, as a result of the blade design and calculation of technological deviations, the option with the best thermo-gas-dynamic characteristics, strength indicators, vibration reliability, and the one taking into account manufacturing errors is selected. At the next level, it can be adjusted using three-dimensional calculation models without losing the indicators of the main selected characteristics. This approach improves the design quality and reduces the time to obtain the best solution.

Keywords*: turbine blades of critical length, formalization, formalization parameters, thermo-gas-dynamics, design, manufacturing technology.*

Introduction

One of the main tasks of the development of large-scale thermal power engineering is to increase the unit capacity of power turbines. Its implementation largely depends on the possibility of creating efficient final stages for steam turbines, the dimensions and design features of which determine the amount of achievable power in one unit, as well as economic indicators and the general structure of the unit (number of cylinders, number of stages, etc.). The creation of such stages is one of the most difficult tasks, since when solving it, contradictions between factors of different nature (thermo-gas-dynamic, structural, strength, technological ones, etc.) become acute. This is especially evident in the design and design of rotor blades. In view of this, the creation of a new stage with a rotor blade of large (critical) length is, as a rule, a new stage in the development of turbine building, the emergence of power units with more modern technological indicators. The contradiction of various requirements and the presence of subjective approaches to solving individual problems in practice turns the design of rotor blades into a complex and sequential process of searching (by trial and error) for a satisfactory solution. A qualitative solution to this problem is possible only with the maximum formalization of the process of creating such blades, which is the basis of the computer-aided design system (CAD). Moreover, a multi-level representation of a technical object (in our case, a blade of a critical length), during the designing of which it is necessary to take into account many factors and parameters, is considered a key element of CAD.

Such an approach at the stage of calculations allows: to systematically use an effective design apparatus; to optimize the resulting solutions at all levels; to apply a developed apparatus of computational mathematics; to significantly reduce the time and resources consumption during design [1].

Currently, there are quite a few turbomachinery design systems, which include blade design, including ANSYS [2, 3].

However, none of the systems fully takes into account the specifics of designing blades of maximum length for the last stage of powerful steam turbines with the formalization of the processes of their creation, in addition, these systems do not include a formalized model of the technological process of blades manufacturing taking into account the fleet of the used machines. However, these factors certainly play an important role in choosing the best blade option, especially when it comes not to modernizing existing stages, but to creating new ones.

The authors of the paper have quite extensive experience in designing and constructing such blades, including the systematization of these processes, on the basis of which the materials of the paper were prepared. The results of the research are given in a number of papers and monographs [4, 5, 6, 7].

The purpose of this paper is to consider the main aspects and features of the methodology for formalizing the process of creating blades of large (critical) length for the last stages of the turbine, taking into account their specifics: significant radial dimensions, high rotation speeds, significant static and dynamic loads.

Taking into account the emergence of new approaches to the design of blades and the turbine flow part, the development of new methods and software packages [8, 9], it is also advisable to summarize the existing experience of formalizing each of the stages of the critical length blade design.

We emphasize that when creating a system for the design and design of blades of the critical length, it is necessary to foresee the presence of models of the thermo-gas-dynamic process; the design process; the technological process of manufacturing, as well as an optimization subsystem and an archive of design solutions. In addition, it is necessary to provide for the calculation of the thermo-gas-dynamic and structural characteristics of the degree, strength and vibration reliability of the rotor blade, gas-dynamic losses in the profile grids, etc.

It seems advisable to have both simplified models (two-dimensional or quasi-three-dimensional), which can be used at the initial stage of design to select the optimal (rational) option of the rotor blade, and more complex three-dimensional models in the system. Such approaches to design with two-level problem solving are used by many authors [10, 11] and show good results.

The quality and reliability of the formalization of processes in all mathematical CAD models plays a decisive role in achieving the effectiveness of the presented approach.

The main aspects and developments of the authors regarding the formalization of mathematical models and the methodology for selecting the optimal (rational) option of the rotor blades of the last stages with the corresponding analysis of the results are given below.

Thermo-gas-dynamic process model

The thermodynamic perfection of the last stage is characterized by the efficiency of energy conversion, i.e. internal efficiency.

It is the internal efficiency of the stage (η_{oi}) that is taken as the dominant quality criterion and is determined in the process of solving algebraic and nonlinear differential equations that describe the spatial flow of the working fluid in the region limited by the radial and axial dimensions of the stage. In this case, both three-dimensional and axisymmetric calculation models can be used.

In general, the dependence of its definition can be presented as follows

$$\eta_{oi}=f(\lambda, X_{\mathrm{td}}, [D]),$$

where λ are output data such as rotor speed, steam flow rate, pressure and enthalpy of steam before the stage, etc.; I_{td} are independent characteristics: distribution of exit angles from the stator and rotor blades $\alpha_1(r)$, $\beta_2(r)$; $[D]$ is the area of determining the dominant quality criterion.

At the first stage of designing and constructing of the blade that is being created, when optimizing the thermo-gas-dynamic characteristics, it is advisable to limit oneself to an axisymmetric calculation with the determination of the flow characteristics along the cross-sections of the inter-crown gaps. This greatly simplifies the solution of the system of equations that formalize the flow process in the stage. The closing system of equations in this case is the distribution of angles $\alpha_1(r)$, $\beta_2(r)$ along the height of the blade (laws of bladed devices twist). These laws significantly affect the level of losses in the grids and losses with the initial speed, and, accordingly, to a large extent determine the value of the internal efficiency of the stage. In addition, they are included in the initial data for profiling and designing of bladed devices and are a connecting link between thermodynamic calculations and the rotor blade design.

As a result of the analysis of the existing practice of designing of the last stages of distributions $\alpha_1=f(r)$ and $\beta_2=f(r)$, the class of these functions and their corresponding analytical dependencies, which are taken as basic at the formalization stage, are defined. In this case, the angle distributions are determined as a function of the relative height of the blade $\bar{l}=l/L$ and a series of coefficients with which it is possible to obtain the entire range of twist laws $\alpha_1(r)$ and $\beta_2(r)$, which actually exist in design practice. In formalized form, these dependencies look like this

$$\alpha_1(\bar{l})=f(\alpha_{1\mathrm{mid}}, \alpha_{1\mathrm{r}}, \alpha_{1\mathrm{p}}, a_1, a_2, \ldots, a_i); \quad \beta_2(\bar{l})=f(\beta_{2\mathrm{r}}, \beta_{2\mathrm{p}}, b_1, b_2, \ldots, b_i), \qquad (1)$$

де $\alpha_{1\mathrm{mid}}$, $\alpha_{1\mathrm{r}}$, $\alpha_{1\mathrm{p}}$ are exit angles from the nozzle diaphragm at the middle one, root and peripheral sections of the blade, respectively; $\beta_{2\mathrm{r}}$, $\beta_{2\mathrm{p}}$ are angles of exit from the rotor blade in the root and peripheral sections, respectively; $a_1, a_2, \ldots, a_i, b_1, b_2, \ldots, b_i$ are coefficients that determine the reference points of the curves that describe the laws of angle distribution along the blade height.

More detailed dependencies for determining $\alpha_1(r)$, $\beta_2(r)$ are given in [5].

The next stage is the parametric formalization stage. The formalization parameters that provide the variation of the angles α_1 and β_2 by the height of the blade are $\alpha_{1\mathrm{mid}}$, $\alpha_{1\mathrm{r}}$, $\alpha_{1\mathrm{p}}$, $a_1, a_2, \ldots, a_i$, $\beta_{2\mathrm{r}}$, $\beta_{2\mathrm{p}}$, $b_1, b_2, \ldots, b_i$. The initial values of these parameters are set within the ranges of their change (restrictions on the formalization parameters).

Also, the ranges of change of the stage characteristics are set at this stage – the area [D]: root reactivity value R_r; restrictions on relative input speed W_1; velocity of the flow exiting the rotor in absolute motion C_2; the presence of convergence in the inter-blade channels; the magnitude of the flow impact at the entrance to the rotor blade, etc.

All these restrictions are determined, as a rule, by concepts arising from the physical nature of the studied thermo-gas-dynamic process and the accumulated knowledge about the creation of blades of the critical length and the corresponding analytical dependencies.

Ensuring these restrictions allows to take into account a number of requirements that are considered as connections that come from other models of the system: design and technological model. It is possible to do as early as at the first stage of design, when the thermo-gas-dynamic characteristics are determined.

Thermodynamic perfection of the stage can be achieved at the next stage - the stage of optimization of a number of its parameters and characteristics.

As a result of the directed search for the optimal solution using appropriate mathematical optimization methods, by varying the formalization parameters taking into account a wide range of constraints, the distributions of angles $\alpha_1(r)$, $\beta_2(r)$ and thermo-gas-dynamic characteristics of the stage, ensuring maximum efficiency, are determined. The use of dependence (1) improves the quality of the optimization process.

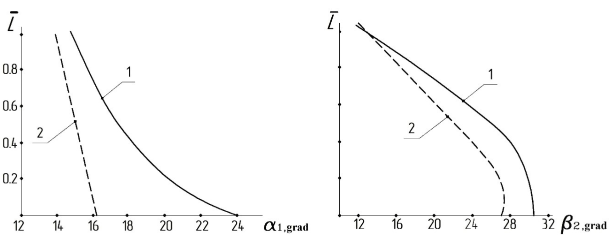
It should be emphasized that the distribution of the constructive angle of entry onto the rotor blade $\beta_1(r)$ either is considered given and is a kind of constraint when searching for optimal twist laws $\alpha_1(r)$, $\beta_2(r)$, or is given approximately, as a first approximation and, being a dependent variable, is refined in the iterative process during the solution of the optimization problem.

The simplicity of the model used at this stage allows to conduct a directed search for the best solution in an automated mode during the optimization process, analyzing hundreds, or even thousands of options. This is especially important when it comes to creating a new blade of the critical length and it is necessary to analyze many blade options, paying special attention to static strength and vibration reliability.

The selected optimal variant of the twist laws of the stator and rotor blades is transferred to the subsystems of the rotor blades profiling and designing. This approach makes it possible to significantly increase the efficiency of the search for the optimal design as a whole, to reduce the design time due to the exclusion of a large number of obviously unsatisfactory option designs from the calculations.

An example the optimal distributions of the angles $\alpha_1(r)$, $\beta_2(r)$, obtained in the process of development and optimization of one of the existing stages with a rotor blade that has a length of L=1030 mm, is shown in Fig. 1.

It should be emphasized that the standard option was selected by its developers as the best one in the result of the analysis of a large number of option studies, and in this case an attempt was made to further improve it using the proposed approaches. Optimization was carried out using the proposed methodology, taking into account the operating modes of the turbine unit. Quality function η_{oi} was defined as additive one, the components of which reflect the efficiency of the stage at the nominal and most representative partial modes with weighting factors corresponding to the number of operating hours of the unit at the corresponding modes (G_i – 1; 0.8; 0.5; t_i – 0.15; 0.8; 0.05, respectively).



a b

Fig. 1. Distribution of flow exit angles from the stator (a) and rotor (b) blades:
1 – standard option; 2 – optimal option

As studies have shown, changing the twist laws of bladed devices taking into account the operating modes of the stage allows to increase the total efficiency of the stage by 1.2%.

At the next stage, it is already possible to carry out optimization calculations in a three-dimensional setting using the results obtained from the axisymmetric model, where the blade option with the best thermo-gas-dynamic, vibrational characteristics and strength is selected as the starting point.

Design process model

At the design stage, the blade is given a shape and dimensions that must ensure the flow path in accordance with the parameters obtained in the process of thermo-gas-dynamic calculations, as well as meet the requirements of technological feasibility, static and dynamic strength.

The most relevant and labor-intensive is the development of methods for optimal design of rotor blades, which include formalized methods for describing the surfaces of the blades, calculations of their stress state, vibration and gas-dynamic characteristics. It is proposed to consider two personal quality criteria at the design stage of the rotor blade – a minimum of integral profile energy losses ξ_b

$$\xi_b = \min\left(1 \Big/ G \int_0^G \xi_{bi} dG_i \right),$$

where ξ_{bi} – are profile energy losses on the i-th cross-section of the blade; G is the steam consumption.

The second criterion is the one that meets the reliability requirements - the maximum separation of the blade's natural frequencies f_i from resonant f_j ones, corresponding to j multiplicity

$$\Delta f_i = \max\{\min\,[f_i - (f_j \pm \Delta f_j)]\},$$

where Δf_j – is the permissible interval to the resonant frequency.

The given personal quality criteria, first of all the second one, should not contradict the dominant, maximum value of the efficiency of the stage, which is achieved by the appropriate profiling of the rotor blades with the obligatory fulfillment of all the basic gas-dynamic requirements imposed on the profile grids. In this case, one should take into account the natural desire of the designer to obtain the largest possible value of the turbine exhaust area, and therefore the size of the stage, in connection with which the rotor blades should be profiled close to equally strong ones, with the achievement of the minimum possible values of centrifugal forces C_b at a given level of the maximum permissible stress σ_t taking into account the material of the blade manufacture.

The minimum possible value of the centrifugal forces of the blade can be determined by the following dependence

$$C_p = F_p[\sigma_t]\exp\left[\frac{k_\delta}{2[\sigma_t]}\left(r_p^2 - r_r^2\right) - 1\right], \qquad (2)$$

where F_{p} is the peripheral cross-sectional area; $k_\delta = \frac{\gamma}{g}\left(\frac{2\pi n}{60}\right)^2 = \frac{\gamma\omega^2}{g}$; g is the gravitational acceleration; γ is the material density; ω is the angular velocity; r_{p}, r_{r} are radii of the root and peripheral sections of the blade, respectively.

By simple transformations of equation (2) and its solution with respect to the length of the blade, we obtain the dependence for determining its maximum dimensions under the condition of static strength

$$L = \sqrt{r_{\text{r}}^2 + \frac{2[\sigma_t]g}{\gamma\omega^2}\left(1 - \ln\frac{F_{\text{p}}}{F_{\text{r}}}\right)} - r_{\text{r}} . \tag{3}$$

In the presence of shelf shrouds and wired connections, equation (3) can be presented in a slightly different form, taking into account statistical data

$$L = \sqrt{r_{\text{r}}^2 + \frac{2g[\sigma_t](1-\lambda)}{\gamma\omega^2}\left(1 - \ln\frac{F_{\text{p}}}{F_{\text{r}}}\right)} - r_{\text{r}} \ , \tag{4}$$

де λ=0 – blade without connections; λ=0,05 – shelf shroud; λ=0,15 – shelf shroud and intermediate connection.

Dependencies (3, 4) clearly show that to obtain blades of maximum length and, as a result, achieve high throughput of the unit at n=const, it is necessary to: reduce the ratio $F_{\text{p}}/F_{\text{r}}$ as much as possible by improving the design of discs capable of carrying large loads; to minimize the number of connections; choose the specific strength of the material σ_t/γ as much as possible.

It should be emphasized that the issues of ensuring the reliability of fixed elements of the nozzle diaphragm, discs, shanks, and shrouds are studied by many authors [8, 9]. Since the last stage operates in the region of low-pressure steam, for fixed elements this task is less complicated than a similar task when creating blades of critical length.

Stress distribution $\sigma_t = f(L)$, which corresponds to dependence (2) (lines 1, 2, 3), is shown in Fig. 2.

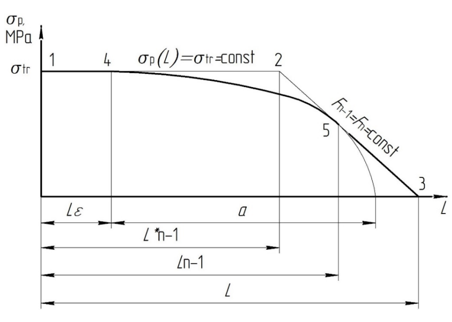


Fig. 2. Tensile stress distribution along the blade height

However, as practice shows, such a distribution with a break negatively affects the smoothness of the blade surface. It is most expedient to choose the law of stress distribution along lines 1, 4, 5, 3 (as shown in Fig. 2) using elliptical curves touching the critical lines. In general, this is represented as

$$\sigma_{ti} = f[D(r), \sigma_{tr}, L_\varepsilon, F_{\text{p}}], \tag{5}$$

where $D(r)$ are blade output data; σ_{tr} is the tensile stress in the root section of the blade; L_ε is the area of an equal-strength blade on which the condition can theoretically be met σ_{tr}=const; F_{p} is the peripheral cross-sectional area.

Further, taking the distribution law $\sigma_t = f(L)$ in analytical form exactly such, i.e. close to the equipotent, as a basis at the formalization stage, areas F_i of the corresponding blade cross-sections can be determined depending on the values σ_{ti} and centrifugal forces of the area in each cross-section, provided that information about the initial peripheral area of the blade is available. In our case we set $F_p = F_{p\text{-}1}$, which corresponds to the desire to achieve minimum centrifugal forces. Thus, taking into account the above in general, the law of area distribution along the radius as a function of the initial data and varying parameters can be presented in the form (analytical dependencies for calculation of σ_{ti} and F_i are given in [4, 5])

$$F(r) = f[\lambda_{\text{r}}(r), \sigma_t(r)] = f[D(r), \sigma_{tr}, F_{\text{p}}], \text{ at } L_\varepsilon = \text{const}.$$

The further task of forming the blade contour is to construct (calculate) the profiles of its cross-sections in direct dependence of the parameters σ_{tr} and F_{p} (parametric formalization stage) on the adopted values. That is, it is necessary to design a profile grid based on the values of the areas previously calculated as a function of the given level of static strength, which provides the flow rotation specified by the results of the thermo-gas-dynamic calculation with minimal energy losses.

It should be noted that the process of formalizing the design of profile grids in the design statement proposed above plays the most important role in forming the surface of a single blade. In view of this, let us briefly dwell on the main features and requirements for the development of such methods. The basis of these methods for their formalization is the idea of the profile configuration, which is based on the application of numerous recommendations, experimental and reference data, for example, it is about ensuring the convergence of the grid channel; smoothness of the profile contours; design of the initial section depending on the flow velocity, etc., which will allow to obtain options with satisfactory gas-dynamic characteristics at this stage. One of the most important requirements is the condition that the calculation of profiles is carried out in a single-valued dependence on the initial data and fixed profile parameters.

Based on the above requirements, the authors have developed methods, for example, using arcs of circles (two- and three-parameter) or using the modified lemniscate equation (three-parameter) – $\rho = a\sqrt{2\left[2\cos^2(\varphi+\gamma)-\lambda\right]}$.

This equation describes curves of different configurations for changes in λ (at λ=1 the equation corresponds to a lemniscate, and at λ=0 – to a circle). These methods have been successfully implemented in the real design of blades of large length [5]. Both the first and second methods allow, unlike many existing ones, to build profile grids in a single way along the entire length of the blade, including, which is very important, at relative grid steps $\bar{t}$=0.25–1.0, which differ significantly from the optimal ones.

Of course, at this stage other parametric methods of profiles design can be used, for example [10, 11, 12], which meet the above requirements.

The third stage of the formalization process (optimization stage) involves the assessment of gas-dynamic and geometric characteristics and the selection of optimal (from the point of view of minimal energy losses) parameters of the profile grids in a wide range of area values and other strength characteristics.

The methods outlined in [4], as well as the capabilities of modern automated systems, allow to present such a variety of profiles with defined parameters and characteristics in the form of nomograms (Fig. 3).

In this figure, the *ABC* curve (k=0.25) corresponds to the grids of two-parameter profiles of the root section with minimal energy losses in the area interval of 17–27 cm^2. Similar nomograms are calculated for all calculated sections.

Next, the procedure for forming the blade surface is presented as follows: for the blade cross-sectional area – F_i, calculated using the given stress law (5), the parameters $X_i(L)$, namely B_i and α_i, are determined uniquely along the line of the gas-dynamic optimum (the *ABC* curve in Fig. 3), with the help of which specific optimal profiles in the calculated sections can be reproduced, and the entire blade surface is formed on their basis in an automated mode (Fig. 4).

It should be noted that the considered links of the subsystem of rotor blades designing, related to the optimization of their gas-dynamic forms, can also be recommended for searching for optimal shapes of stator blades.

Based on the material presented above, it can be stated that the considered formalized methods of profile grids designing allow, in accordance with the initial data $D(r)$ and the basic parameters (in our case σ_t, F_p), to uniquely determine the effective profiles in the calculated blade cross-sections. Based on this, the expression for the formalized blade shape in general form can be written as

$$\Phi=f[D(r), \sigma_{tri}, F_{pi}]=f[D(r), F_i(L)]=f[D(r), X_i(L)]. \qquad (6)$$

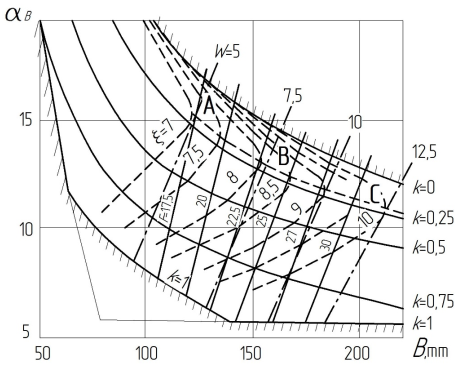


Fig. 3. Nomogram of profiles of the root section of the blade:
α_B – profile installation angle;
B – profile width;
F_i – profile area in the i-th section, cm^2;
ξ – gas-dynamic energy losses, %;
k – flow inflow coefficient;
W – profile resistance moment, cm^3

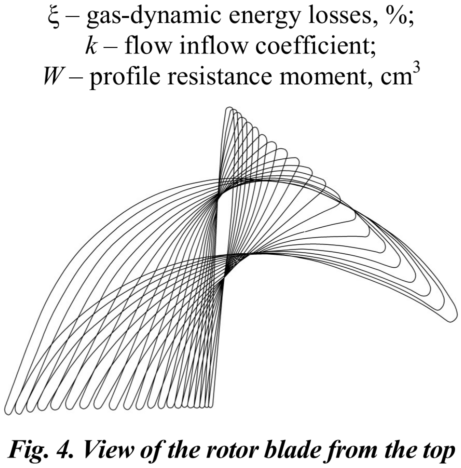


Fig. 4. View of the rotor blade from the top

The advantage of the presented formalized approach to blade design is the ability to describe the entire blade surface in analytical form as a function of the initial data and parameters σ_{fr}, F_{p}. This allows, at the optimization stage, to obtain many options of blade designs with a full package of their characteristics (static and dynamic strength, stage efficiency, cost, etc.), which are stored in the archive of centrifugal forces design solutions, by varying the selected parameters. The capabilities of the system and the used methods are described in detail in [5]. They allow to visualize the state of the design with various combinations of a moderate number of parameters and to build an array of blades in the form of a nomogram, as shown in Fig. 5.

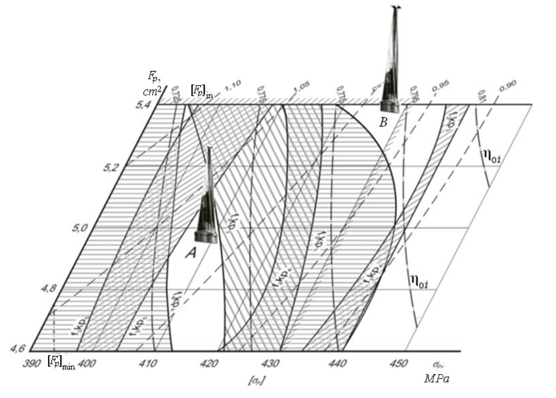


Fig. 5. Nomogram of rotor blades (L=1030mm):
F_{p} and σ_t – varying parameters;
$[F_{\text{p}}]_3$, $[F]_{\min}$, $[\sigma_t]$ – technological, gas-dynamic and strength limitations, respectively;
η_{oi} – efficiency of the stage;
C_c – relative manufacturing cost;
f_{kp} – natural frequency of oscillations

Each point of such a nomogram corresponds to a specific blade with certain characteristics, the shaded areas are unacceptable vibration zones.

It is obvious that zone "B" on the diagram is the best according to the main quality criteria (efficiency, maximum distance from resonant frequencies, cost). The optimal (rational) design should be considered a design with parameters σ_t=430 MPa, F_{p}=5.4 cm^2, which corresponds to zone "B". According to the above algorithm (6), it is not difficult to restore the design of the blade that meets these basic parameters (Fig. 5, option *B*).

At the final stage of the rotor blade design process, it is necessary to perform a three-dimensional calculation of the thermo-gas-dynamic characteristics of the stage taking into account the obtained option of the rational blade. In this case, if necessary, the correction of the blade surface (profiles) is carried out in such a way as to maintain the level of static strength, which means maintaining the previously calculated values of the cross-sectional areas F_i. In this case, centrifugal forces do not change, and the vibration characteristics, if changed, change insignificantly within the zone of permissible restrictions.

Based on the methodology for effective formalization of the rotor blade creation process considered in the paper, the designers of JSC "Ukrainian Energy Machines" (until recently JSC "Turboatom") have designed and manufactured a promising rotor blade made of steel of the critical length (1100 mm), which allows to significantly increase the unit unitary power, or significantly reduce the metal content of the turbine plant.

Process model

Since technological operations are an integral part of the entire process of creating bladed devices and can very noticeably affect the main characteristics of the stage, there is an urgent need to formalize this production stage, including it with all its connections in the general system of searching for the optimal (rational) option of the rotor blade. First of all, it is necessary to consider the issue of the degree of influence of technological errors on the main indicators of the efficiency and reliability of the rotor blades, since, as studies show, even small errors can lead to significant changes in some characteristics of the stages [13].

Deviations corresponding to a specific technological process can be assessed using statistical and probabilistic methods if there is a sufficient amount of statistical information.

Currently, there is significant technological progress in the manufacture of rotor elements and, above all, blades of turbines for various purposes (steam, gas, aviation, etc.) [14, 15]. However, in the production of large blades of powerful steam turbines, 4–5-spindle milling machines from FOREST are most often used, as before [16].

In this paper, a series of rotor blades with a length of 960 mm was selected as the object of research. Their finishing milling was carried out on the above-mentioned machine.

During the research, about 10,000 measurements were performed on a series of manufactured blades, which were carried out using a special built-in probe. Similar procedures can also be carried out using 3D scanning systems. However, the final result of the research is not affected by how information about technological deviations from the project was obtained.

To process a large volume of measurements, their systematization and analysis, a special program has been developed that allows to visualize information about the random technological error of the surface of a real turbine blade in a form familiar to a designer or technologist.

The numerical studies were carried out using this program and the CAD program "Last Stage" [5], which provides the possibility of calculating the thermo-gas-dynamic and strength characteristics of the stage taking into account technological deviations in the blades production.

The quantitative assessment of the influence of technological deviations on the effective characteristics of the blades was carried out using the example of the blade considered above. Analysis of the obtained data showed that their influence is quite significant and cannot be neglected. Thus, the centrifugal force spread was ΔC_b=40 kN, and compared to the design blade ΔC_b=90 kN, the bending stress from the steam load – $\Delta\sigma_b$=15 MPa, (20 MPa), total stresses – $\Delta\sigma_\Sigma$=20 MPa (44 MPa), natural oscillation frequencies Δf_1=6 Hz (7.5 Hz), Δf_2=13 Hz (15 Hz), Δf_3=12.3 Hz (24 Hz). The deviations from the design blade are given in brackets.

The impact of a slight change in the shape of the blade on the efficiency of the stage was also assessed. The efficiency of the stage with average deviations of the rotor blade differed from the design by $\Delta\eta_{oi}$=0.35%, and the option with the maximum deviation – by 0.43%. The increase in losses was most sensitive to deviations from the calculated thicknesses of the leading edge (δ_e) and the cross-section throat (δ_t).

The main practical conclusion that follows from the conducted research is that even small deviations in the shape and size of the profiles of the rotor part of the blades from the theoretical ones, which occur during any technological process of blades manufacturing, affect both the strength and economic indicators of the stage. Moreover, as calculations have shown, even deviations that vary within the accepted tolerances are noticeable. In view of this, the task of the researcher, designer and technologist is to find such methods and technological processes that would minimize these effects [16], or take them into account at the stage of turbomachine stages designing, which requires the creation of appropriate mathematical models for assessing technological errors.

In this regard, the possibility of creating mathematical models based on experimental and statistical material that link the geometric dimensions of the blade with the main factors of manufacturing and design is of great interest.

Such formalized approaches can be implemented by probabilistic methods that allow for error prediction using a minimum amount of information about the blade configuration, machine tool data, reference materials, and available experimental data. In particular, one of these relations [17] is obtained from the condition of the probability theory theorem on the addition of variance under the assumption that the distribution of total errors is carried out according to a normal law under the assumption of independence of the factors under consideration from each other, and with the equality of the adjustment size with the design values of the blade coordinates

$$\partial_{\Sigma ij} = K_1 t^* / 2\sqrt{\lambda^*_{1ij}\delta^*_{1ij} + \lambda^*_{2ij}\delta^*_{2ij} + \ldots + \lambda^*_{nij}\delta^*_{nij}} + K_2 \Delta C^*_{ij}\,, \qquad (7)$$

where λ^* is the relative scattering coefficient; $\delta_{\Sigma ij}$ is the total linear error of the j-th blade section of the i-th profile point; K is the empirical coefficient that corrects the calculated dependence taking into account experimental data; δ^*_{ij} are elementary random errors; t^* is the coefficient set depending on the percentage of taken risk; ΔC^* are systematic errors.

This analytical dependence (7) is basic in our further research. With its help, the possible maximum deviations from the standard size during machining can be determined, provided that the total errors are normally distributed. In general, the total deviations from the design ones can be written as follows

$$\Phi\delta_\Sigma = f(\textstyle\sum\delta_{ij}, \Delta C). \qquad (8)$$

Next, we will analyze the possibility of representing dependence (8) in a parametric form with the determination of the dominant factors influencing the errors (parametric formalization stage).

Random errors are formed mainly taking into account reference (passport) materials and a priori dependencies, taking into account statistical dependencies, and systematic errors are formed from a priori analytical dependencies. For example, errors from tool wear δ_w and temperature deformations δ_t can be determined as follows: $\delta_w = a_1 l/L$; $\delta_t = b_0 + b_1 l/L$, where a_1, b_0, b_1 – regression coefficients.

As production experience has shown, the largest share (up to 60%) in the processing of large blades is made up of systematic errors, which are most often determined by elastic and residual deformations of the workpiece δ_{im} (errors from residual deformations were not considered in the paper).

To determine the printing errors, an a priori dependence of the form can be used

$$\delta_{\text{im}} = C_0 + C_1 l/L + C_2 (l/L)^2 + C_3 (\delta_{\text{pr}}/\delta_{\max}) + C_4 (\delta_{\text{pr}}/\delta_{\max})^2 + C_5 (l/L)(\delta_{\text{pr}}/\delta_{\max}),$$

where δ_{pr}, $\delta_{\max}$ – current and maximum thickness of the cross-section profile that can be milled; C_i – regression coefficients.

However, the greatest interest is the possibility of a more accurate numerical determination of the systematic error in functions of the geometric cross-sections of the blade. Let us consider this issue in more detail. The value of the quantity $\Delta C' \approx \delta_{\text{im}}$, as practice shows, mainly depends on the stiffness of the blade, least of all on the tool. The absolute values of the deflections of the part (elastic deformations) are considered as a function of the force arising as a result of milling and the stiffness of the blade. Therefore, the mathematical model of the technological operation should include in this case an algorithm for calculating the stiffness of the analyzed structure under the conditions of a given technological process and the values of compliance from the action of a unitary force at characteristic points of the blade cross-section profile along its height. To analyze the stressed state of turbomachine blades, taking into account the influence of static loads, a set of programs developed by the IPMach of the NAS of Ukraine based on the finite element method was used [18].

Calculations using these programs allow to obtain a "compliance array", i.e. the values of displacements at characteristic points of the blade when 1 kgf is applied to this section. As a result, the problem of determining the error of the imprint δ_{im} is reduced to calculating the value of the effective cutting force, which depends primarily on the processing modes and the design of the blade surface. The circumferential component of the cutting force P_z with sufficient accuracy for our studies can be calculated [19] according to the following empirical dependence:

$$P_z = C_p t^{xp} S_z Z_{\text{cut}} B_{\text{cut}}^{R_p} D_{\text{cut}}^{Z_p} X_p, \tag{9}$$

where C_p, K_p are empirical coefficients depending on the material of the part; S_z is the feed per one cutter tooth; Z_{cut} is the number of cutter teeth; B_{cut} is the milling width; D_{cut} is the cutter diameter; X_p, Y_p, R_p, Z_p are degree indicators that depend on the material of the part.

Taking into account the above, with known values of the "compliance arrays", dependence (9) allows to determine the possible magnitude of the systematic error at each characteristic point of the blade with fixed cutting parameters and tabular (passport) information about the maximum errors of the machine tool and its accessories.

The obtained calculation results are shown in Fig. 6.

From Fig. 6 it is seen that the values of the total errors calculated by the finite element method (curve 2) are quite close to the indicators of static deviations of a real blade.

Thus, for practical use, based on numerous calculations, the dependence (7), which includes a probabilistic estimate of random errors and a numerical method for determining systematic deviations, is recommended.

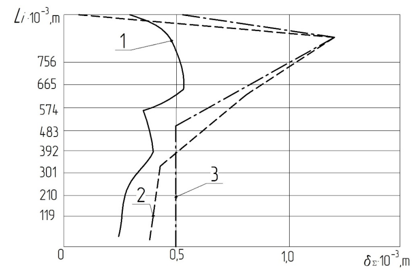


Fig. 6. Distribution of total errors:
1 – ΔC determined a priori by formula 13;
2 – calculated by the finite element method under the influence of cutting force;
3 – statistical data of the real blade

So, the dependence for determining the error during blade manufacturing $\Phi_{\delta\Sigma}$ in general can be represented as follows:

$$\Phi_{\delta\Sigma} = f(\sum \delta_{ij}, \Delta C) = f(\sum \delta_{ij}, S_z, B_{\text{cut}}, D_{\text{cut}}, Z_{\text{cut}}).$$

In our case, at this stage of parameterization, for greater clarity and simplicity, the parameters S_z and B_{cut} were taken as the most dominant.

The developed means of calculating technological models of the formation of the blade surface as a result of manufacturing errors make it possible to set the task of optimizing cutting modes at the design and technological preparation stage in order to determine both tolerances for deviations of linear dimensions, taking into account their impact on operational characteristics, and rational processing parameters within the framework of the specified design tolerances and other restrictions (optimization stage).

Since at the stage of selecting rational modes, the quality function is the cost-effectiveness equivalent to productivity (The full cost estimate for the production of the blade is not considered in this paper), then the optimization in this option can be based on the function of determining the time for finishing the blade T during milling, which is found as the ratio of the path length L_{ni} to the feed of S_m: $T=L_{ni}/S_m$.

When determining the path length, B – a cubic spline, was used. With the help of this spline the contours of the intermediate sections of the blades were established with a step equal to the milling width B_{cut}.

Variation of the mode parameters (S_m, B_{cut}) causes changes in the cutting forces at the point of its application on the blade and the magnitude of the total error, the change of which occurs primarily due to the change in the systematic error δ_{im}.

Below, on the example of a blade with L=960 mm, in the form of a nomogram (Fig. 7) in the feed coordinates of S_m and the processing time T, the results of the study of all the varying modes are given. Here, the mode with the parameters S_m=300 mm/min, B_{cut}=7.5 mm, T=90 min (point A), which correspond to the processing time, feed, milling width, which are inherent in the real technological process of finishing milling of blades, as well as the limitations and the quality function, is also plotted (stage efficiency). It should be emphasized that it is the connection through the database with the CAD that allows to assess the impact of errors on the operational characteristics of the blade, including efficiency.

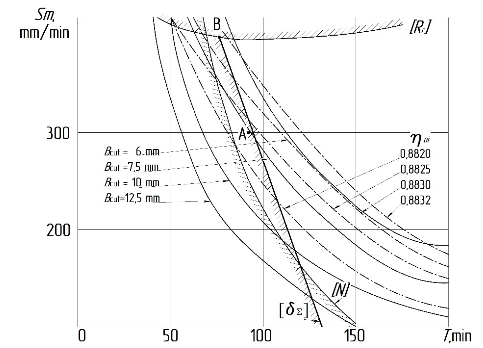


Fig. 7. Diagram of technological blades: S_m and B_{cut} – varying parameters (feed and width of the cutter); [δ_Σ] – design tolerance, T – processing time; [N] – limitation on drive power; [R_r] – permissible surface roughness; η_{oi} – efficiency of the stage

As a result of using the presented approach of theoretical determination of errors and analysis of the relations between mode parameters, limitations and productivity, it is possible to choose more rational parameters, for example: B_{cut}=6 mm; T=80 min, S_m=400 mm/min. This leads to a reduction in the processing time of one blade by 10 minutes compared to the real technological process. This is 6% lower in cost than the regulated option. At the same time, such a mode (p. B, Fig. 7) in contrast to the initial one (p. A, Fig. 7), provides the level of permissible errors.

The considered approach allows the designer and technologist to effectively analyze the entire process of creating turbine blades (design + manufacturing), since there is a real possibility of assessing the influence of modes and productivity of blade machining on the reliability and efficiency of the stage. At the same time, such a relation allows to take into account the features of the technological process at the stage of the blade surface shaping and, if necessary, to make the appropriate correction either by cutting modes or by changing the profile contour (without reducing the efficiency of its flow around it).

Conclusion

The methodology for formalizing the processes of creating blades of the last stage of maximum length, taking into account the features of blade design, including the technology of its manufacture, is presented. The feasibility and necessity of creating a system for automated design of blades of the critical length are shown, taking into account the features of design and manufacture, compliance with the hierarchy of mathematical models, direct and feedback relations between models. Based on the conducted research, general approaches to the effective formalization of mathematical models inherent in each stage of the last stage design are defined (marked):

– justified choice of the basic dominant condition and the basic analytical function for each of the models of the subsystem under consideration;

– parametric formalization of all modeling analytical dependencies included in the overall design process, with maximum use of accumulated experience, recommendations and reference material;

– optimization of the parameters and characteristics of the last stage at each stage of its creation, taking into account a wide range of constraints.

When formalizing the thermo-gas-dynamic model, the possibility of considering all existing twist laws of blade devices when searching for an optimal solution was taken into account, which significantly increases the quality of the optimization process.

The main advantage of the proposed formalization at the design stage is the ability to analyze a large array of blade options that meet the technical task, taking into account all limitations and information about all the main characteristics of each blade. The information is given in the form of nomograms that allow the designer to choose one of the best options from a variety of options.

A formalized probabilistic mathematical model, adjusted taking into account real statistical data, which describes technological deviations on the tops of blades with satisfactory reliability for practical calculations, taking into account the processing modes used in finish milling, has been developed. This allows the designer, at the stage of searching for the optimal solution, to consider the design not as ideal, but close to the real one, which is obtained as a result of the calculation within the framework of the design and technological models, to evaluate and take into account the influence of manufacturing errors on the strength and gas-dynamic characteristics of the blade, as well as on the efficiency of the stage as a whole. In addition, it becomes possible to carry out a rational choice of processing parameters in the process of technological preparation, and at the design stage, when shaping the blade surface, to take into account the specific features of the machine tool equipment.

A two-level approach to solving the problem, when at the first level, during optimization using simpler mathematical models, the best one is selected from a set of options, which can be adjusted (or further improved) at the next level, is provided.

The considered methodology for formalizing the processes of creating rotor blades of the critical length allows to obtain optimal solutions when designing new blades, taking into account the specifics of their creation and a wide range of limitations.

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Ефективна формалізація процесів проектування – як основний фактор у досягненні оптимальних рішень при створенні останніх ступенів парових турбін

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На основі наявного досвіду проєктування й конструювання лопаток останнього ступеня великої (граничної) довжини й аналізу літературних джерел встановлені особливості методології формалізації процесів створення таких лопаток з урахуванням їх специфічних особливостей (великих радіальних розмірів, неоптимальних відносних шагів решітки $\bar{t}$ *=0,25–1,0, високих статичних і динамічних навантажень). Представлено параметричну формалізацію основних моделюючих залежностей процесів, на яких базується створення робочих лопаток: термогазодинамічного процесу, конструювання лопатки й технологічного процесу виготовлення. Обґрунтовано необхідність створення систем (підсистем) автоматизованого проєктування лопаток великої довжини з наявністю в системі моделі технологічного процесу виготовлення лопатки. Вона базується на висновках про те, що навіть невеликі відхилення від проєктного варіанта в межах допуску при виготовленні лопатки впливають на термогазодинамічні характеристики ступеня, особливо якщо йдеться про горлові перерізи. Розроблено формалізовану ймовірнісно-статистичну математичну модель, яка дозволяє описати технологічні відхилення поверхонь лопаток з урахуванням режимів обробки, що використовуються при чистовому фрезеруванні із задовільною для практичних розрахунків достовірністю. Це дає змогу вже на етапі проєктування взяти до уваги вплив похибок виготовлення і специфічних особливостей верстатного обладнання на показники міцності лопатки, її газодинамічні характеристики, а також на ефективність роботи ступеня. Запропоновано дворівневий підхід до процесу проєктування, що дозволяє за* *двомірною моделлю вести спрямований пошук найкращого рішення в автоматизованому режимі, аналізуючи сотні варіантів з урахуванням широкого кола обмежень. У подальшому в результаті конструювання лопатки й розрахунку технологічних відхилень обирається варіант із найкращими термогазодинамічними характеристиками, показниками міцності, вібраційної надійності й з урахуванням похибок виготовлення. На наступному рівні він може буди скоригований з використанням тривимірних розрахункових моделей без втрати показників основних обраних характеристик. Такий підхід підвищує якість проєктування і скорочує час отримання найкращого рішення.*

***Ключові слова**: турбінні лопатки граничної довжини, формалізація, параметри формалізації, термогазодинаміка, конструювання, технологія виготовлення.*

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