DOI: https://doi.org/10.15407/pmach2026.01.047

UDC 621.787:519.87

MATHEMATICAL MODELING AND ANALYSIS OF SURFACE ROUGHNESS FORMATION DURING VIBRATION-CENTRIFUGAL HARDENING BASED ON MULTI-FACTOR EXPERIMENTATION

Ivan V. Klymash
ivan.v.klymash@lpnu.ua ,
ORCID: 0009-0006-2384-3539

Lviv Polytechnic
National University,
12, S. Bandery str., Lviv,
79013, Ukraine

Improvement of the operational reliability of critical machine parts is largely determined by the surface layer state formed during finishing operations. In this regard, research into surface plastic deformation processes, which combine structural strengthening with the achievement of minimum surface roughness, is of high relevance. The process of surface roughness formation in 30KhGSA steel during vibration-centrifugal hardening using fixed profiled rollers is studied in this paper. In contrast to processing in a loose abrasive medium, this approach ensures a deterministic character of the process and technological inheritance of the tool geometry on parts with stress concentrators. The aim of the paper is to establish quantitative regularities of the influence of technological factors: processing time (t), vibration amplitude (A), and working gap (Z) on the arithmetic mean deviation of the profile (R_a). To solve this problem, a full factorial experiment of the 2^3 type with logarithmic transformation of input variables, which allowed for the linearization of the power model and ensured high approximation accuracy, was applied. Statistical analysis using Cochran's, Student's, and Fisher's criteria confirmed the model's adequacy and revealed that the working gap (Z) is the dominant factor. A negative effect of excessive processing duration (over 8 min) was identified, leading to an increase in R_a due to micro-fatigue failure and the overshoot (surface peeling) phenomenon. Using the Box-Wilson steepest ascent method, optimal modes were determined, ensuring a reduction in roughness from 6.45 μm to a predicted level of 1.68 μm. The resulting model (R=0.998) possesses high predictive capability and can serve as a mathematical foundation for algorithmizing finishing hardening operations and developing systems for technological production planning. The obtained results provide a basis for justifying the rational operating regimes of vibrational-centrifugal hardening with profiled tools, ensuring the formation of a stable microrelief and the induction of compressive residual stresses. This enhances the operational durability and fatigue strength of parts containing stress concentrators.

Keywords: *vibration-centrifugal hardening, surface roughness, full factorial experiment, mathematical modeling, regression equation, Box-Wilson method, optimization of technological regimes.*

Introduction

Ensuring the operational reliability of critical machine parts operating under cyclic loads is one of the priority tasks of modern mechanical engineering [1]. It is known that fatigue failure usually occurs in stress concentration zones, such as the recesses of threaded profiles and technological grooves. Traditional methods of blade and abrasive processing do not always allow to form a surface layer with the necessary physical and mechanical properties, since they are often accompanied by residual tensile stresses and microgeometric defects [2]. In this regard, methods of surface plastic deformation are widely used, which provide structural strengthening, increase microhardness and create fields of compressive residual stresses that block the development of microcracks [3, 4]. Among the methods of the surface plastic deformation (SPD), a special place is occupied by vibration-centrifugal hardening. This process is based on a synergistic combination of a low-frequency vibration field and centrifugal forces, which allows to intensify plastic deformation and ensure the formation of a regular microrelief [5]. The parameters of dynamic interaction (kinetic impact energy, frequency, working gap) are decisive for the formation of a regular microrelief and ensuring a given surface quality [6]. Mathematical modeling of such processes makes it possible to predict the state of the surface layer at the design stage [7]. Special attention is required to study the strengthening regimes for structural alloyed steels, in particular, 30KhGSA steel. The high initial hardness and specific carbide heterogeneity of this material create the risk of defects such as "overshoot" or microcracks with excessively intensive processing regimes [8].

Analysis of research and publications

The fundamental works of leading scientists are devoted to the study of the physicochemical foundations, dynamics and technological capabilities of the vibration-centrifugal hardening (VCH). In particular, the mechanisms of formation of an ultrafine-grained structure and gradient phase states in the surface layer, which provide a significant increase in the fatigue strength and wear resistance of parts, have been studied in detail [4, 9]. It has been established that under conditions of intense plastic deformation during VCH, the grain size can decrease to 19–43 nm, which is accompanied by an increase in the microhardness of the material [8]. Previous studies of the VCH process, carried out on KhVG tool steel, demonstrated the significant potential of the method in increasing operational characteristics: an increase in the microhardness of the surface layer by 31.8% compared to the initial state was recorded. This confirms that the dynamic impact action during VCH provides not only the formation of the relief, but also a deep structural modification of the material.

The possibility of forming a reinforced layer with a significant depth (up to 6 mm) and a high level of compressive residual stresses reaching 1600 MPa has been studied [2, 6]. The constructed models are based on the principles of functionally oriented design, which allows technologically ensure the specified quality parameters.

However, the analysis of existing publications shows that most of the developed mathematical models are focused on the processes of machining parts in free abrasive or hardening media. In such models, roughness is considered as a result of multiple chaotic micro-impacts of the medium particles, the motion of which is often described through the dynamics of a "pseudo-gas" [7, 10]. In contrast to such machining in a free abrasive, the use of fixed profiled rollers makes the process deterministic. This provides technological inheritance of the tool geometry on parts with stress concentrators, which is a key advantage of this paper, since it allows predictably form the parameters of the microrelief.

The specifics of local strengthening of the recesses of threaded grooves and technological hubs require the use of profiled rollers fixed to the device. In this case:

1) the process becomes deterministic, i.e. the tool geometry is copied to the part profile according to the principles of technological inheritance [2];

2) the dynamics of the system is determined not only by the vibration parameters, but also by the rigidity of the tool system and the size of the working gap (Z), which acts as the main regulator of the kinetic energy of the impact.

This necessitates the refinement of the mathematical apparatus for predicting the arithmetic mean deviation of the profile R_a as a function of dynamic factors specifically for circuits with a discrete profiled contact, which is considered in this paper.

Aim and objectives of the study

The aim of the paper is to establish quantitative patterns of the influence of key technological factors of vibration-centrifugal hardening: processing time (t), vibration amplitude (A) and working gap (Z), on the formation of the arithmetic mean deviation of the profile R_a of parts made of 30KhGSA steel when using profiled rollers, as well as to substantiate a rational processing trajectory to minimize surface roughness. To achieve the set goal, it is necessary to solve the following tasks:

– to determine statistically significant technological factors and assess the nature of their mutual influence on the surface quality parameters under the conditions of shock-impulse action of rollers;

– to build an adequate mathematical model in the form of a regression equation, which allows to predict the level of roughness R_a depending on the dynamic operating modes of the system;

– to carry out a technological interpretation of the physical influence of each parameter on the state of the surface layer, in particular, the role of the working gap (Z) as the main regulator of the kinetic energy of hardening;

– to develop practical recommendations for the selection of processing modes for parts with stress concentrators based on the Box-Wilson steepest ascent method, ensuring the achievement of minimal roughness without the risk of overshoot defects.

Materials and research methods

Methodology and experimental design. For effective study of complex multifactorial processes, among which vibration-centrifugal strengthening is recognized, it is important to use a formalized approach to experimental planning. In this study, the methodology of a full factorial experiment (FFE) of the 2^3 type was used, which allows to obtain the maximum amount of information about the influence of each factor and their interactions with the minimum required number of experiments ($N=8$), ensuring high reliability and validity of the conclusions.

Experimental setup and object of study. The object of study were samples of 30KhGSA steel (Fig. 1), which is characterized by high hardness.

The mechanism of the VCH plant operation. Experimental studies were conducted on a specialized vibrating pendulum semi-automatic machine (Fig. 2). The design of the machine ensures operation in a resonant mode, which guarantees high stability of the amplitude characteristics of the process. The key feature of the system is the transformation of the kinetic energy of the machine into a localized strengthening effect on the surface of the stress concentrator. The energy source is an asynchronous motor (1.7 kW), which drives a two-section unbalanced shaft through a belt drive.

The hardening process is carried out using an experimental device (Fig. 3) fixed on the workbench. During the rotation of the shaft, complex spatial elliptical trajectories of the table, which are transformed into directed planetary kinematics of the roller assembly, are generated. This provides a high frequency of shock pulses directly at the root of the concentrator. Under the action of vibration-centrifugal force, three profiled rollers inertially move within the working gap in the direction of the part. Due to the high hardness of the profiled roller (62 HRC) and the exact coincidence of the roller profiles and the threaded groove, the impact energy is focused on the bottom of the concentrator. This causes intense surface plastic deformation, which leads to nanostructuring of grains and the formation of a stable layer of residual compressive stresses. After the impact, due to a change in the table acceleration vector, the roller moves back. Free rotation of the upper part of the device ensures a gradual rotation of the rollers around the part. Each subsequent blow falls on a new point along the circumference of the groove, ensuring the continuity of the reinforced layer.

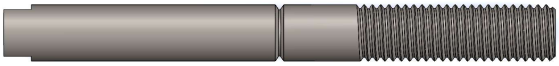

Fig. 1. Sample with localized stress concentrator (razor-shaped groove)

Fig. 2. General view of a pendulum semi-automatic machine for vibration-centrifugal hardening:
1 – engine; 2 – unbalanced shaft; 3 – workbench;
4 – experimental device

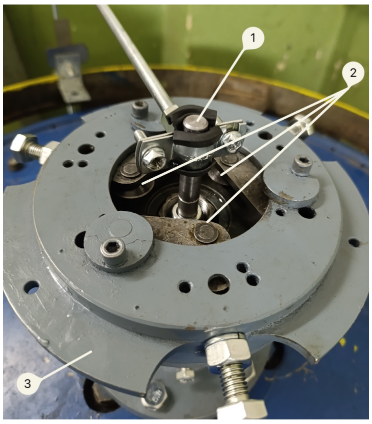

Fig. 3. Experimental device mounted on the VCH plant:
1 – sample; 2 – rollers with a threaded profile;
3 – device body

Identification of response factors and parameters. The following variables were identified during the study.

Independent factors (process input parameters):

– processing time (t, min) – duration of shock-pulse action;

– vibration amplitude (A, mm) – range of vibrations of the device part;

– working gap (Z, mm) – distance between the roller and the bottom of the groove, which determines the acceleration energy of the tool.

Dependent parameter (response parameter): arithmetic mean deviation of the profile (R_a, μm) is a key roughness indicator that characterizes the quality of the processed surface.

Method of measuring the arithmetic mean deviation of the R_a profile. To ensure precision measurement accuracy in the hard-to-reach area of the root of the threaded groove, a non-contact interference 3D profilometer "Micron alpha" was used. The choice of the white light interferometry method allowed to avoid errors inherent in probe methods, where the radius of the needle often exceeds the radius of the microprofile cavity. The measurement process included vertical scanning with a lens to record the moments of maximum interference contrast, on the basis of which a digital height map was built. The value of R_a was calculated by the software as the arithmetic mean of the absolute deviations of the profile points from the average line on the baseline. For each of the 8 experiments, two parallel measurements were carried out at different points along the groove according to the planning matrix.

Levels of variation of factors and coding of variables. Since the relation between roughness parameters and SPD modes is usually power-law

$$R_a = C_0 \cdot t^m \cdot A^n \cdot Z^p ,$$

then logarithmic transformation of input variables was applied for linearization of the model. The transition from natural values (*t, A, Z*) to the coded dimensionless variables (X_1, X_2, X_3) was carried out by logarithmizing the natural values of the factors according to the limits of variation given in Table 1. For each factor, two levels of variation were selected: lower (-1) and upper (+1), which determine the limits of the studied range.

Table 1. Variation levels and intervals

Factor	Symbol	Natural units	Lower (-1)	Upper (+1)	Basic level (X^0_i)	Variation interval (ΔX_i)
Time, min	X_1	min	2	8	5	3
Amplitude, mm	X_2	mm	1	5	3	2
Gap, mm	X_3	mm	4	10	7	3

Experimental data and construction of a mathematical model

The VCG treatment was carried out for 30KhGSA steel with the following factor values: time (*t*) 2–8 min, amplitude (*A*) 1–5 mm, working gap (*Z*) 4–10 mm. To ensure high reproducibility and the possibility of estimating the experimental error, each experiment was carried out twice (r=2). The obtained roughness values R_a for each experiment are given in Table 2.

Table 2. Output data

Experiment No.	Time (*t*)	Amplitude (*A*)	Working gap (*Z*)	R_{a1}	R_{a2}
1	8	5	10	8.437	6.365
2	2	5	10	1.331	0.803
3	8	1	10	3.038	3.620
4	2	1	10	0.832	1.056
5	8	5	4	29.682	26.631
6	2	5	4	13.013	11.800
7	8	1	4	41.742	55.936
8	2	1	4	8.916	6.442

Construction of a mathematical model and statistical verification of its adequacy. The complete planning matrix, including coded and natural factor values, as well as the results of roughness measurements, is presented in Table 3.

Table 3. Experiment planning matrix

Experiment No.	X_1		X_2		X_3		R_{a1}, µm	ln R_{a1}	R_{a2}, µm	ln R_{a2}	$\overline{R}_a$, µm	$\overline{Y}_v = \ln \overline{R}_a$
	code	*t*, min	code	*A*, mm	code	*Z*, mm						
1	+1	8	+1	5	+1	10	8.437	2.133	6.365	1.851	7.401	1.992
2	-1	2	+1	5	+1	10	1.331	0.286	0.803	-0.219	1.067	0.034
3	+1	8	-1	1	+1	10	3.038	1.111	3.620	1.286	3.329	1.199
4	-1	2	-1	1	+1	10	0.832	-0.184	1.056	0.054	0.944	-0.065
5	+1	8	+1	5	-1	4	29.682	3.391	26.631	3.282	28.157	3.337
6	-1	2	+1	5	-1	4	13.013	2.566	11.800	2.468	12.407	2.517
7	+1	8	-1	1	-1	4	41.742	3.732	55.936	4.024	48.839	3.878
8	-1	2	-1	1	-1	4	8.916	2.188	6.442	1.863	7.679	2.026

Based on the planning matrix, regression coefficients were calculated. Regression equation with coded variables, taking into account the interaction of factors

$$Y = b_0 + b_1 X_1 + b_2 X_2 + b_3 X_3 + b_{12} X_1 X_2 + b_{13} X_1 X_3 + b_{23} X_2 X_3 + b_{123} X_1 X_2 X_3 . \quad (1)$$

The general form of the regression equation in coded dimensionless variables, taking into account all interaction effects, has the form

$$Y = 1{,}865 + 0{,}737 X_1 + 0{,}105 X_2 - 1{,}075 X_3 - 0{,}042 X_1 X_2 + 0{,}068 X_1 X_3 + 0{,}117 X_2 X_3 + 0{,}215 X_1 X_2 X_3 .$$

Statistical analysis and model optimization. To confirm the reliability of the obtained results, a comprehensive statistical analysis was conducted.

1. Checking the homogeneity of variances: the calculation according to the Cochran's C test showed that G=0.402. Since the obtained value is less than the critical one (G<0.6798 for α=0.05), the hypothesis of homogeneity of variances is accepted. This indicates the high quality of the experiments.

2. Assessment of the significance of the coefficients: according to the Student's t-test, the confidence interval Δb_i=0.1155 was determined. The coefficients b_0, b_1, b_3, b_{23} and b_{123} were found to be significant. The coefficient b_2=0.105, although it is less than the significance threshold, was left in the model to preserve the physical description of the influence of the vibration amplitude on the strengthening process. The terms b_{12} and b_{13} were recognized as insignificant and removed from the model.

3. Adequacy of the model: Fisher's exact test confirmed the adequacy of the constructed equation. The calculated value F=1.303 is significantly less than the tabulated F_{kp}=4.46, which allows to use the model for forecasting.

4. Approximation accuracy: the calculated multiple correlation coefficient R=0.9981 indicates that the obtained mathematical model describes the real physical processes of microrelief formation of 30KhGSA steel during VCH by 99.8%.

Mathematical model in natural variables. After carrying out the potentiation procedure and transition from logarithmic codes to physical parameters (t, A, Z), the empirical dependence for determining the arithmetic mean deviation of the R_a (μm) profile took the following form

$$\overline{R}_a = 28.48 \cdot t^{2.31} \cdot A^{1.68} \cdot Z^{(-1.66 - 0.67 \ln t - 0.84 \ln A + 0.83 \ln t \ln A)} \cdot e^{-1.53 \ln t \ln A} .$$

Analysis and discussion of results

Analysis of the resulting model allows to understand how each technological parameter and their combinations affect the final surface quality, measured by the roughness R_a. The high predictive ability of the model (R=0.9981) is due to the deterministic nature of the interaction: unlike stochastic processes in a free environment, where chaotic microshocks prevail, the use of a profiled tool guarantees a clear copying of the profile according to the principle of technological inheritance. This allows to control the formation of R_a with high accuracy through the parameters of the gap, time and amplitude.

Ranking the absolute values of the regression coefficients (1) allows to establish a hierarchy of the influence of factors on the formation of roughness:

– dominant influence of the working gap (Z): the coefficient b_3= -1.075 is the largest in modulus, which determines the gap as the most influential factor in the studied range. The negative sign clearly indicates that increasing the gap (up to 10 mm) leads to a significant decrease in roughness. Physically, this is explained by the increase in the amplitude space for the acceleration of the rollers, which allows the tool to accumulate more kinetic energy before impact, providing more intensive smoothing of the initial irregularities;

– influence of the processing time (t): the coefficient b_1=0.737 is the second largest and has a positive sign. This means that the increase in roughness (deterioration of quality) occurs with an increase in the processing duration beyond optimal values. Such an effect may be associated with the phenomena of surface layer fatigue or the accumulation of microdamages with excessively long impact action. The obtained results on the change in roughness correlate with the physics of steel hardening during SPD. It should be noted that under similar processing regimes of KhVG tool steel, an increase in microhardness, which indicates intensive plastic deformation, was achieved. For the studied steel 30KhGSA, this explains the nature of the change in R_a: initial intensive smoothing is accompanied by an increase in hardness, however, when the energy threshold is exceeded (time over 8 min), the plasticity resource is exhausted, which leads to microfatigue fracture;

– the influence of the vibration amplitude (A): the coefficient b_2=0.105 is the smallest among the linear effects. This indicates that the independent influence of the amplitude on the roughness within the studied values (from 1 to 5 mm) is less significant compared to the processing time and the gap.

The complex nonlinear nature of the VCH process is confirmed by the significance of the interaction effects (b_{23}=0.117 and b_{123}=0.215). This means that the influence of each factor dynamically changes depending on the levels of other parameters, which makes the use of the resulting regression equation necessary for accurate process control.

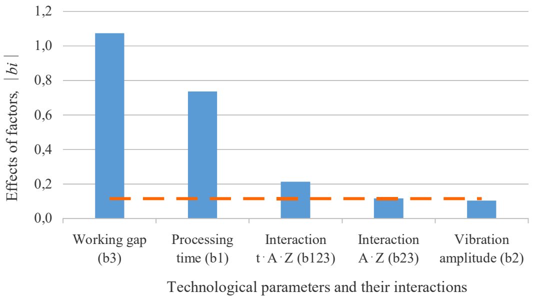

Fig. 4. Pareto chart of the influence of technological factors on roughness

For hierarchical ranking of the degree of influence of the studied factors on the surface quality, a Pareto chart was constructed (Fig. 4).

In the processes of vibration-centrifugal hardening, the arithmetic mean deviation of the surface profile (R_a) and its stress state are interrelated results of the transformation of the kinetic energy of tool impacts into the work of plastic deformation. This relation is based on such key mechanisms as:

– energy distribution: the kinetic energy of roller impacts is spent simultaneously on two processes – smoothing of initial micro-roughnesses (reduction of R_a) and accumulation of plastic deformation in the surface and subsurface layers;

– compensation of micro-stress concentrators: profile irregularities act as micro-stress concentrators, where the theoretical concentration coefficient increases in proportion to the height of irregularities. Residual compressive stresses create the effect of "pre-loading". This generates a "closing" force on potential fatigue cracks at the bottom of micro-depressions in the profile, which increases fatigue strength;

– influence of processing modes and the effect of overshoot: increasing the working gap (Z) to 10 mm expands the space for roller acceleration, which provides higher impact energy, more intensive surface smoothing and a simultaneous increase in the level of compressive stresses. However, the increase in R_a detected in the experiment with a processing duration of more than 8 min indicates the exhaustion of the material's plasticity resource. This effect, known as "overshoot", is accompanied by micro-fatigue failure and relaxation (reduction) of compressive stresses due to loss of metal integrity.

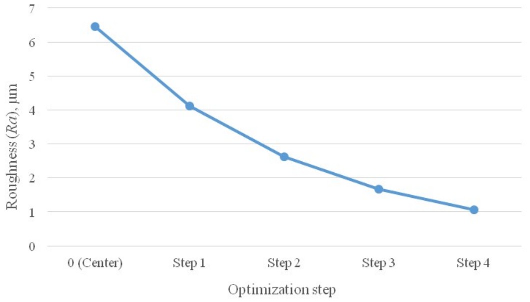

Fig.5. Response surface and projection of Box-Wilson steepest ascent steps for the VCH process

Process optimization using the Box-Wilson method. To minimize the arithmetic mean deviation of the *Ra* profile, the steepest ascent method was used (Fig. 5), which allows for a systematic analysis of complex interrelations of factors and finding the extremum with a minimum number of control experiments. The direction of movement to the optimum was determined by the signs of the linear coefficients of the obtained equation (1): the processing time (t) and the amplitude (A) were subject to decrease, and the working gap (Z) was subject to increase. The gap (Z) with a working gap ΔZ=1.0 mm was chosen as the basic factor. Proportional steps for other factors were calculated based on the ratio of their coefficients to b_3.

Building a trajectory of movement to the extremum. The calculation of the imaginary experiments was carried out from the center of the experiment (zero level point: t=5 min; A=3 mm; Z=7 mm) along the gradient vector. The results of the roughness prediction are given in Table 4.

Table 4. Results of roughness prediction using the steepest ascent method

Experiment No.	Time (t), min	Amplitude (A), mm	Working gap (Z), mm	Forecast R_a, µm
Center (0)	5.00	3.00	7.0	6.45
Step 1	4.32	2.93	8.0	4.12
Step 2	3.64	2.86	9.0	2.63
Step 3	2.96	2.79	10.0	1.68
Step 4	2.28	2.72	11.0	1.07

The use of the steepest ascent method allowed to identify a rational processing trajectory. Analysis of the predicted values shows that the "Step 3" mode (gap 10 mm; time 3 min) is optimal, as it provides a reduction in roughness from 6.45 μm to 1.68 μm without the risk of destruction of the structure of 30KhGSA steel. Further increase in impact energy ("Step 4") may be inappropriate due to the likelihood of the "overshoot" effect, which requires additional verification by second-order models (CCD) for a precise description of the extremum zone. Thus, the applied Box-Wilson method serves as a tool for quickly entering the zone of rational modes, providing the foundation for further precision automation of the VCH process.

Conclusions

1. Based on the 2^3 type FFE, an adequate mathematical model of the formation of roughness Ra during vibration-centrifugal hardening of 30KhGSA steel has been constructed. The high correlation coefficient (R=0.9981) confirms that the model describes real physical processes in the studied range of factors by 99.8%.

2. A hierarchy of the influence of technological factors on the surface quality has been established: the dominant one is the working gap (Z), the increase of which to 10 mm provides intensive smoothing of micro-unevenness due to the rational distribution of the kinetic energy of impacts. The time factor (t) is of critical importance due to the risk of overshoot defects when the processing duration exceeds 8 min.

3. The effects of the interaction of factors (b_{23}, b_{123}) were quantitatively evaluated, which proves the nonlinear nature of the VCH process and the need to use higher-order models for accurate prediction of the state of the surface layer.

4. The use of the Box-Wilson steepest ascent method allowed to justify a rational processing mode: gap 10 mm, time 3 min, amplitude 2.8 mm. This ensures a reduction in the arithmetic mean deviation of the R_a profile from 6.45 μm to 1.68 μm, which meets the requirements for finishing operations of critical parts.

5. The results of the study are recommended for use in methods of functionally-oriented design of technological processes for the final hardening of parts made of special steels.

References

1. Skoczylas, A. & Zaleski, K. (2022). Study on the surface layer properties and fatigue life of a workpiece machined by centrifugal shot peening and burnishing. *Materials*, vol. 15, iss. 19, article 6677. https://doi.org/10.3390/ma15196677.
2. Kusyi, Ya., Lychak, O., Topilnytskyy, V., & Bošanský, M. (2019). Development of the finishing and strengthening technological operations using SADT-technologies. *Ukrainian Journal of Mechanical Engineering and Materials Science*, vol. 5, no. 3–4, pp. 57–69. https://doi.org/10.23939/ujmems2019.03-04.057.
3. Kyryliv, Ya., Maksymiv, O., Kyryliv, V., Tsizh, B., Vynar, V., & Yemelyanenko, S. (2025). Impact of deformation force by vibration-centrifugal hardening on the properties of the surface nanostructure of steel 40Kh. *Journal of Materials Engineering and Performance*, vol. 34, pp. 29182–29192. https://doi.org/10.1007/s11665-025-11450-9.
4. Kyryliv, Ya., Kyryliv, V., Sas, N., & Dutka, V. (2020). Residual stresses formed by vibration-centrifugal hardening. *Advances in Materials Science and Engineering*, vol. 2020, article 5189473, 7 p. https://doi.org/10.1155/2020/5189473.
5. Stupnytskyy, V., Kusyi, Ya., Dragašius, E., Baskutis, S, & Chatys, R. (2023). Modeling of vibrational-centrifugal strengthening for functional surfaces of machine parts. In: Tonkonogyi, V., Ivanov, V., Trojanowska, J., Oborskyi, G., Pavlenko, I. (eds.) Advanced Manufacturing Processes V. InterPartner 2023. *Lecture Notes in Mechanical Engineering*. Springer, Cham, pp. 231-244. https://doi.org/10.1007/978-3-031-42778-7_21.
6. Halan, Yu. Ya. (2021). *Pidvyshchennia efektyvnosti tekhnolohichnoho protsesu vibratsiino-vidtsentrovoyi obrobky detalei v sypuchomu abrazyvnomu seredovyshchi* [Improvement of technological process efficiency of vibration-centrifugal processing of parts in a loose abrasive medium]. PhD dissertation. Ternopil: TNTU, 194 p. (in Ukrainian).
7. Lyashuk, O., Mytnyk, M., Aulin, V., Lutsiv, I., Tkachenko, I., Galan, Yu., Perenchuk, O., & Kondratiuk, O. (2024). Mathematical model of vibration-centrifugal processing of parts using loose abrasive. *Key Engineering Materials*, vol. 981, pp. 87-106. https://doi.org/10.4028/p-WHu4HR.
8. Hevko, B. M., Kondratiuk, O. M., Liashchuk, O. L., Marchuk, M. M., Serilko, L. S., & Halan, Yu. Ya. (2017). Pidvyshchennia efektyvnosti vibratsiino-vidtsentrovoho obroblennia detalei vilnymy abrazyvamy [Increasing the efficiency of vibration-centrifugal processing of parts with free abrasives]. Rivne: National University of Water Management and Environmental Management, 136 p. (in Ukrainian).
9. Kyryliv, V. I., Kyryliv, Ya. B., & Sas, N. (2018). Formation of surface ultrafine grain structure and their physical and mechanical characteristics using vibration-centrifugal hardening. *Advances in Materials Science and Engineering*, article 3152170, 7 p. https://doi.org/10.1155/2018/3152170.
10. Kundrák, J., Mitsyk, A. V., Fedorovich, V. A., Markopoulos, A. P., Grabchenko, A. I., & Kundrák, J. (2021). Simulation of the circulating motion of the working medium and metal removal during multi-energy processing. *Machines*, vol. 9, iss. 6, article 118, 22 p. https://doi.org/10.3390/machines9060118.

Received 04 March 2026 *Accepted 19 March 2026* *Published 30 March 2026*

Математичне моделювання та аналіз формування шорсткості поверхні при вібраційно-відцентровому зміцненні на основі багатофакторного експерименту

І. В. Климаш

Національний університет «Львівська політехніка», 79013, Україна, м. Львів, вул. С. Бандери, 12

Підвищення експлуатаційної надійності відповідальних деталей машин значною мірою визначається станом поверхневого шару, що формується на фінішних операціях. У зв'язку з цим актуальним є дослідження процесів поверхневого пластичного деформування, які дозволяють поєднати зміцнення структури із забезпеченням мінімальної шорсткості поверхні. У роботі досліджено процес формування шорсткості поверхні сталі 30ХГСА при вібраційно-відцентровому зміцненні із використанням закріплених профільованих роликів. На відміну від обробки у вільному абразиві, такий підхід забезпечує детермінований характер процесу й технологічне успадкування геометрії інструмента на деталі з концентраторами напружень. Метою роботи є встановлення кількісних закономірностей впливу технологічних чинників: часу обробки (t), амплітуди коливань (A) й робочого зазору (Z) на середньоарифметичне відхилення профілю R_a. *Для вирішення задачі застосовано методологію повного факторного експерименту типу* 2^3 *із логарифмічним перетворенням вхідних змінних, що дозволило лінеаризувати степеневу модель і забезпечити високу точність апроксимації. Статистичний аналіз за критеріями Кохрена, Стьюдента та Фішера підтвердив адекватність моделі й допоміг виявити, що домінуючим чинником виступає робочий зазор (Z). Виявлено негативний ефект надмірної тривалості обробки (понад 8 хв для сталі даного класу), що призводить до зростання* R_a *внаслідок мікровтомного руйнування й явища перенаклепу поверхневого шару. За допомогою методу крутого сходження Бокса-Вілсона визначено оптимальну траєкторію руху у просторі факторів, яка дозволяє забезпечити зниження шорсткості з 6,45 мкм до прогнозованого рівня 1,68 мкм. Отримана модель (R=0,998) має високу прогностичну здатність і може бути використана як математичне підґрунтя для алгоритмізації фінішних операцій зміцнення й розробки систем технологічної підготовки виробництва. Отримані результати дозволяють обґрунтувати раціональні режими вібраційно-відцентрового зміцнення профільованим інструментом, що забезпечує формування стабільного мікрорельєфу і створення залишкових напружень стиску. Це сприяє підвищенню експлуатаційної довговічності й втомної міцності деталей із концентраторами напружень.*

***Ключові слова**: вібраційно-відцентрове зміцнення, шорсткість поверхні, повний факторний експеримент, математичне моделювання, рівняння регресії, метод Бокса-Вілсона, оптимізація технологічних режимів.*

Література

1. Skoczylas A., Zaleski K. Study on the surface layer properties and fatigue life of a workpiece machined by centrifugal shot peening and burnishing. *Materials*. 2022. Vol. 15. Iss. 19. Article 6677. https://doi.org/10.3390/ma15196677.
2. Kusyi Ya., Lychak O., Topilnytskyy V., Bošanský M. Development of the finishing and strengthening technological operations using SADT-technologies. *Ukrainian Journal of Mechanical Engineering and Materials Science*. 2019. Vol. 5. No. 3–4. P. 57–69. https://doi.org/10.23939/ujmems2019.03-04.057.
3. Kyryliv Ya., Maksymiv O., Kyryliv V., Tsizh B., Vynar V., Yemelyanenko S. Impact of deformation force by vibration-centrifugal hardening on the properties of the surface nanostructure of steel 40Kh. *Journal of Materials Engineering and Performance*. 2025. Vol. 34. P. 29182–29192. https://doi.org/10.1007/s11665-025-11450-9.
4. Kyryliv Ya., Kyryliv V., Sas N., Dutka V. Residual stresses formed by vibration-centrifugal hardening. *Advances in Materials Science and Engineering*. 2020. Vol. 2020. Article 5189473. 7 p. https://doi.org/10.1155/2020/5189473.
5. Stupnytskyy V., Kusyi Ya., Dragašius E., Baskutis S, Chatys R. Modeling of vibrational-centrifugal strengthening for functional surfaces of machine parts. In: Tonkonogyi V., Ivanov V., Trojanowska J., Oborskyi G., Pavlenko I. (eds.) Advanced Manufacturing Processes V. InterPartner 2023. *Lecture Notes in Mechanical Engineering*. Springer, Cham, 2023. P. 231-244. https://doi.org/10.1007/978-3-031-42778-7_21.
6. Галан Ю. Я. Підвищення ефективності технологічного процесу вібраційно-відцентрової обробки деталей в сипучому абразивному середовищі: дис. … д-ра філ. … 131 «Прикладна механіка». Тернопіль : ТНТУ, 2021. 194 с.
7. Lyashuk O., Mytnyk M., Aulin V., Lutsiv I., Tkachenko I., Galan Yu., Perenchuk O., Kondratiuk O. Mathematical model of vibration-centrifugal processing of parts using loose abrasive. *Key Engineering Materials*. 2024. Vol. 981. P. 87-106. https://doi.org/10.4028/p-WHu4HR.
8. Гевко Б. М., Кондратюк О. М., Лящук О. Л., Марчук М. М., Серілко Л. С., Галан Ю. Я. Підвищення ефективності вібраційно-відцентрового оброблення деталей вільними абразивами. Рівне: НУВГП, 2017. 136 с.
9. Kyryliv V. I., Kyryliv Ya. B., Sas N. Formation of surface ultrafine grain structure and their physical and mechanical characteristics using vibration-centrifugal hardening. *Advances in Materials Science and Engineering*. 2018. Article 3152170. 7 p. https://doi.org/10.1155/2018/3152170.
10. Kundrák J., Mitsyk A. V., Fedorovich V. A., Markopoulos A. P., Grabchenko A. I., Kundrák J. Simulation of the circulating motion of the working medium and metal removal during multi-energy processing. *Machines*. 2021. Vol. 9. Iss. 6. Article 118. 22 p. https://doi.org/10.3390/machines9060118.